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ABSTRACT

This document consists of two issues of the journal "Seedbed," an outcome of the Teachers' Center Project at Southern Illinois University at Edwardsville. Designed to be neither scholarly nor trivial, each issue is a collection of teachers' ideas that they thought worth sharing with other teachers as "seedlings" to be developed, helping teachers to construct a point of view toward the intellectual growth of children or promoting children's thoughtful probing of mathematical ideas. Issue Number 27 contains 31 articles, including some which share ideas and reactions to a week spent by teachers in Yorkshire, England, schools. Number 28 contains 38 articles with an array of activities plus comments on children's educational growth and well being. (PK)

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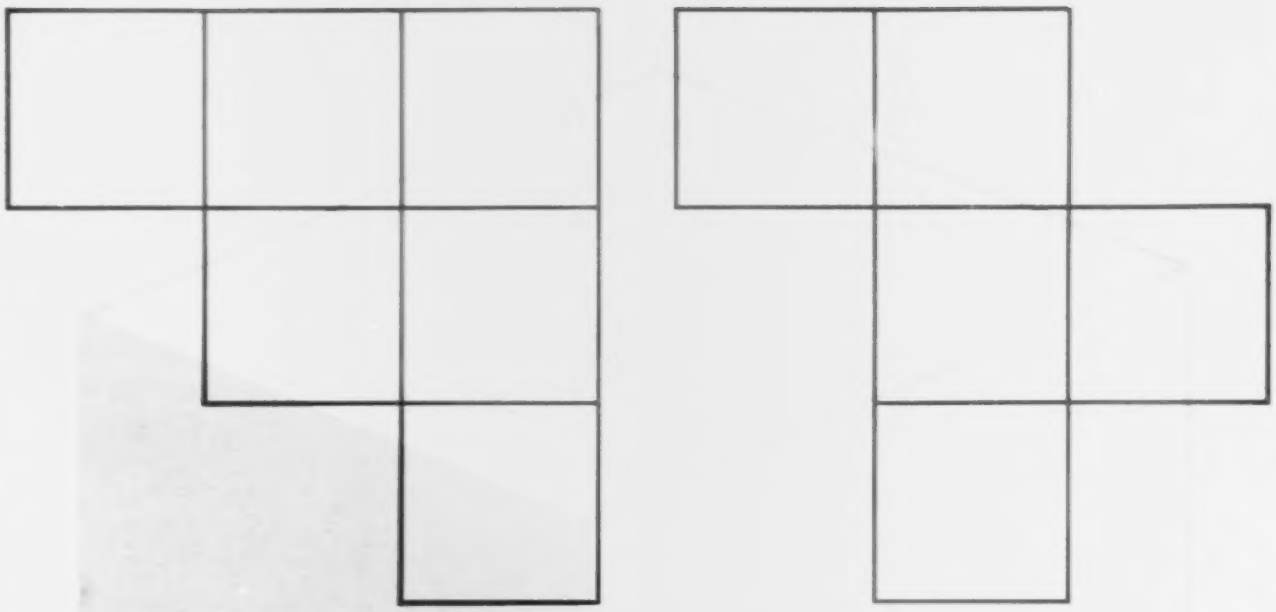

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SEEDBED

NUMBER 27

MAY 1987

Teachers' Center Project

SIUE

Southern Illinois University at Edwardsville

INTRODUCTION

Although technology has once again reared its ugly head to make the delivery a difficult one (transporting text from an Apple][E to a Macintosh causes headaches), we are proud and happy to bring you two new issues of **SEEDBED**.

SEEDBED 27 is due in large measure to the innovation and insights of David Lingard, head teacher of Northcliff Comprehensive School, Yorkshire, England, and Sylvia Johnson, senior lecturer in Maths Education at the Sheffield Polytechnic, Sheffield, England, and of course, Professor T. C. O'Brien, director of the Teachers' Center Project at SIUE. It is my hope that some of the sparks of their creativity will light the fires of many more teachers -- for education is not a spectator sport.

My special thanks to the many "techy" people who worked on our computer problems and at long last found a solution. They showed us problem solving in the real world. Thanks too must go to Laurie Nelson, word processor extrordinaire, and to Angie Fuller, who was always ready to pitch in when needed.

The cover art is inspired by some of the problems presented by David and Sylvia. These are detailed in "Summer 1986: An Experience In Problem Solving" found in this issue.

Read, enjoy, and continue to share the seeds of learning with your colleagues.

Barbara Goldenhersh
Editor

P.S. GOOD NEWS!! SEEDBED is now available on microfiche through Educational Resources Information Center (**ERIC**), Columbus, Ohio.

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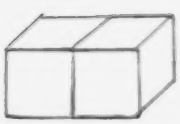
SUMMER 1986: AN EXPERIENCE IN PROBLEM SOLVING

Barbara Goldenhersh

A class with David Lingard and Sylvia Johnson is an experience of immersion into problem solving at its best. One becomes engrossed in an idea, a thought, which keeps you awake nights as you try to work it through to solution--or to an additional "what if". Putting the problems to print must lose much in the translation. The body language, the enthusiasm of the group and the wonderfully accepting, non-judgmental way in which David and Sylvia presented ideas and offered responses which led us on, rather than offering answers, cannot be conveyed. Yet, it is important that the problems be presented so that you, the reader, can work with them, experience them, offer them to children, and elaborate on them yourself. Remember the by words -- How else can you..., what other ways..., are you sure that's the only answer...???

What follows is an attempt to convey many of the problems offered during the 1986 Teachers' Center Summer Course.

1. Given 5 cubes, how many different ways can they be placed? They must be placed full face to face--that is:



Two models are the same if one can be turned or flipped to make the other.

   a. How many ways can 2 blocks be placed? 3 blocks? 4? 6? Is there a pattern?

2. Using any number of cubes placed in a square or rectangle, what is the size of the enclosure, i.e. the amount of fencing?

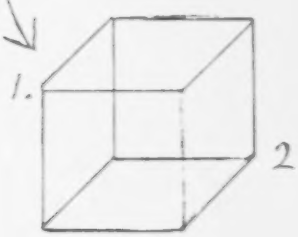
   a. Does the same number of blocks placed differently equal the same number inside? Why or why not? Is there a pattern?

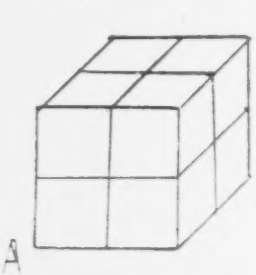
   b. How does change in the number of blocks *in* the rectangle affect this?

What can you deduce?

3. Make a cube from 12 construction paper strips using glue or tape. (The strips should be folded lengthwise for strength and attached at the corners.) A cube should be available for every 2 children.
   THE PROBLEM: An ant is on the struts of the cube at point 1. It wants to get to point 2. What is the shortest number of struts it can follow to get there? How many ways can it get there? Record each way in some manner. Walking a strut only once, what are the most struts it can use? Record all the ways the ant can travel from point 1 to point 2.

   a. If some have reached closure and are able to justify their discovery, add the idea of a cluster of cubes. How many moves from point A to B?





4. Most are familiar with the way to fold a cube

Can a cube be formed using , or ,

or

How many ways can you find? Children can experiment with this by taping square cards into different patterns and attempting to make a cube.

5. Given the numbers 22 -> 14 -> 10 -> 8 -> __ , what number will come next? Why? You may come up with a number of responses. The pattern continues 22 -> 14 -> 10 -> 8 -> 7-> 1. Does this fit your idea? In this case, the rule we are thinking of has to do with factoring.

22	->	14	->	10	->	8	->	7	->	1
1,2,11,22		1,2,7,14		1,2,5,10		1,2,4,8		1,7		1

Eliminating the number itself, the factors add up to the next number. Thus 22 becomes 1+2+11=14. From this many questions can be asked.

a. What number comes before 22? Is there more than one answer?

b. Do all numbers lead to 1? If not, what else could happen?

c. Consider all numbers from 1 to 50 for example. How many chains? Which number is the most steps/jumps from 1?

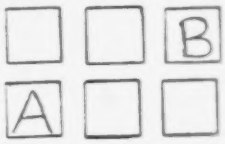
d. Are there numbers to which no number leads?

6. Here's another direction for factor chains. Take only the prime numbers from the factors and add these. The factors for 12 are 1, 2, 3, 4, 6. Since 2, and 3 are prime, 20 -> 5

or

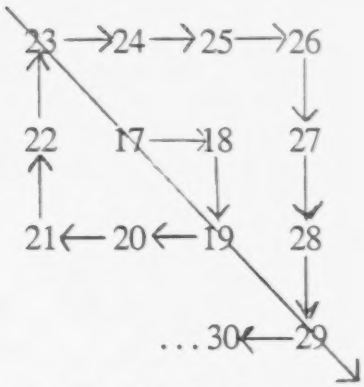
Take prime factors which multiply to the number. For example, 2 x 2 x 3 = 12. Adding the unique numbers, the prime numbers 2 + 3 = 5 so 12 -> 5. 20 = 2 x 2 x 5 and 2 + 5 = 7, so 20 -> 7.

7. Place yourself in the problem. Make two rows of three chairs. Five chairs have people in them. In these rows of "airline seats", the person at A (an aisle seat) wants to move to B (a window seat).



Moves can only be made vertically or horizontally and must be made to an empty seat. No seats may be jumped. How many moves will it take for A to get to B? What is the least number of moves? Add more chairs. (3x3) What is the least number of moves? 3x4? 4x4? Do all the people move? Do they all move the same number of times? How many times do they move?

8. Start with 17. Make a number spiral



Look at the numbers on the main diagonal. Are they primes forever?

9. Draw one dot on paper. Draw a circle which has the dot on the circumference. Draw another circle, and another. How many could you draw? Draw two dots. Draw a circle with both dots on the line. Draw another. How many can you draw? Draw 3 dots. Draw a circle with all three points on the circumference. How many can you draw? Can you make three dots through which no circle can be made? Draw four dots.

Make a circle. Can you draw a circle through four points? When can you? When can't you?

10. Draw three circles. These can touch or not touch and they can be any size. Describe them to someone and have them recorded on the board. Determine which are repetitive of other patterns and remove the duplicates. How many ways can you place them? By what rules? (Paper circles can be used for younger children.)

11. Take a circle. Make 2 cuts (straight lines all the way through). How many regions can you get? Make three cuts. How many regions can you get? What is the most? What is the least? Make 4 cuts. (At this point it becomes difficult to do without drawing.) What is the most you can get? What is the least? Keep going. Are there patterns?

12. As in # 11, where you cut a circle, do the same using a three dimensional figure. What is the largest number of pieces you can end up with?

13. Place three circles with a radius of 1" so all are touching. What is the area of space in between?

14. Make a paper circle and cut out a section. Make a cone out of it and fill it with rice or sand. If a bigger section is cut out, will it hold more or less? What cone will hold the most -- have the biggest volume? What cone the least.

15. A rope is placed around the earth at the equator so it fits tightly. It is removed, lengthened 10 meters and replaced. What amount of gap is there all around between the rope and the earth?

16. A spin off on factor chains. How many ways of making 5?

    1+1+1+1+1=5

1+1+1+2=5

1+1+3=5

1+4=5

1+2+2=5

2+3=5

What about other numbers? 3? 4? 6?

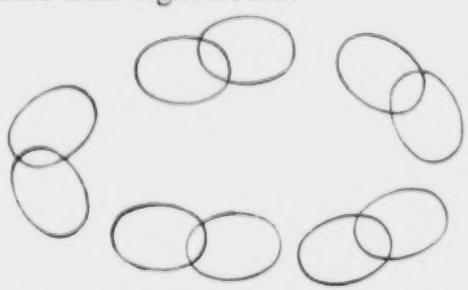
Make a chart -- Number	Number of ways to get it
3	2
4	4
5	6
6	10

Any patterns here?

How many ways of making 10? 12? 100?

17. You have a four minute timer and a seven minute timer. How would you measure nine minutes?

18. A chain has broken into five pieces of two links each. It takes one hour to cut through a link and one hour to weld it. How can the length of chain be repaired in less than eight hours?



19. There are ten piles of money consisting of gold coins. All the coins in one pile (and only one pile) are fake but we don't know which pile. You know

the correct weight of a legal coin and you know that a fake coin weighs one gram less than the gold coins. How can you identify the pile of fakes by only one weighing of coins?

20. A 40 pound weight has broken into four pieces. Luckily, the pieces can be used to weigh any amount in whole pounds from 1 to 40 pounds. How much does each piece weigh? One can precede this exercise by seeing what 2 weights can measure, what three weights can measure, and what would be the best.

21. Guess My Rule

YES	NO

Students call out numbers to fit a rule which you have made (but not disclosed). Put their guesses in the correct column, yes or no. The first to guess the rule comes up to lead. What's a possible rule? Numbers divisible by 9. Rules can have more than one variable. The possibilities are endless. This game can be used for other learning areas such as grammar, history, etc.

22. Everyone sits in a circle. A die or ball is handed around the circle to every second person. Will it arrive back at the start the first go round? the second? the third? What if you skip to the third person? The fourth person? etc. What difference will the number of people in the circle make?

23. Everyone sits in a circle. Each person makes up a rule to use when they toss the bean bag -- ie. 1 to the left, 2 to the right, people wearing green, boys. Anyone can stop the game to guess another's rule. If they are wrong, they are out. If they are right the person whose rule was guessed is out.

24. Area: Ask pairs of children to create a specific construction paper shape which has a perimeter of 20 inches. Some examples are square, triangle,

circle, rectangle, diamond, rhombus, L-shape. Place these on the board in random order. Each makes their own guess as to which has the smallest and largest area. Someone comes to the board and places the shapes in what they perceive to be the order from smallest to largest. Others should be allowed to challenge. The challenger must support his or her position.

By the way, how many rectangles can you make with a perimeter of twenty? Order these in regard to area. Is there a pattern?

25. Geoboards: Place a single rubber band around 8 pins on a geoboard. Note the number of pins inside and bordering the shape and the area of that shape. Record as shown for each shape you form.

Pins on boundary	Pins inside	Area of shape (number of squares)
8	0	3
4	4	5
7	1	3 1/2
5	3	4 1/2 etc.

Can you see a pattern for increases and decreases in area? If so, does it carry through for other numbers?

Given an area, can you tell how many pins are on the boundary and how many inside? Is there more than one possibility?

26. Can you fit 6 different shapes into a geoboard area 4 pins by 8 pins? Is there more than one solution? Dot paper or graph paper can be used if geoboards are not available.

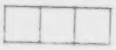
27. How many pennies will fit around a penny so they touch? How many fit on the next ring around? And the next?

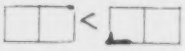
    a. How many pennies will fit around a half dollar?

b. What happens with other sized circles?

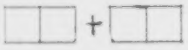
28. Dice games. A die is thrown and with each throw children put the number into one of the boxes to try for a particular outcome. For example, with two throws, make the smallest possible number.

How about these:

a. the largest number, 

b. a true sentence, 

c. number closest to 250,

d. the smallest total, 

e. some of your own.

29. This dice game is played in pairs. Two dice and 10 colored chips are needed for each player. Place any numbers from 2 to 12 in a 3x3 grid. Each player throws two dice and a counter is put on the sum. The object is to fill the grid with your color (if one player is playing) or to make a row vertically, horizontally, or diagonally. There can be two colors on a square (if more than one player.

Variations: Only one chip on a square allowing for blocking, or, if the same sum is rolled, the opponents chip can be knocked off. The grid can be enlarged to 4 by 4 or 5 by 5 (for 3 dice).

30. Another dice game played in pairs. The same equipment is needed as above. Put the numbers 1-6 in a 4x4 grid (the numbers will need to be repeated.) The dice are thrown and added. Chips can be placed on the numbers thrown or on other numbers which add to the same sum.

Variation: Multiply the thrown dice. Cover two numbers which multiply

to that sum.

31. Domino activities. Arrange a double six set of dominoes to make a circle which matches up all the way around. Can it be done? If so, can you remove one (not a double) and still match them up? Can you remove all the sixes and match them up? Why or Why not? Can you remove all the fours and match them up? Why?

32. Use all the dominoes to make sums. One will be left over. Can you make a sum that includes the extra domino?

    Variations:

    a. Place the dominoes horizontally to make 2 digit numbers and make sums.

    b. Do as above using subtraction.

33. Turn dominoes to the blank side. Fit dominoes together to make a solid form. The line the arrows point to is called a fault line. That is, you can get straight through the rectangle on that line. By adding additional dominoes, can you make a rectangle which does not have a fault line? What size rectangles will work? Can you make a square? In what sized squares?

34. Group problem solving activity. The class is divided into small groups (in this case, groups of 4). Each group is given information for the problem (see the attached sheets for a sample problem) but they must share the information to solve it.

    Having done the problem, have the participants think about how the group worked by listing the following questions.

    1. Did your group have a leader?

2. How was the leader chosen/selected/identified?

3. Did you want to be the leader?

4. Did you get cross/angry with, or frustrated by your leader?

5. Did anyone dominate/try to dominate your group?

6. How did you try to handle this problem?

7. Was anyone left out of your group's work?

8. How did you try and integrate that person?

9. Did anyone in the group act as scribe

10. Did you work effectively as a group?

    If so, what helped that happen?

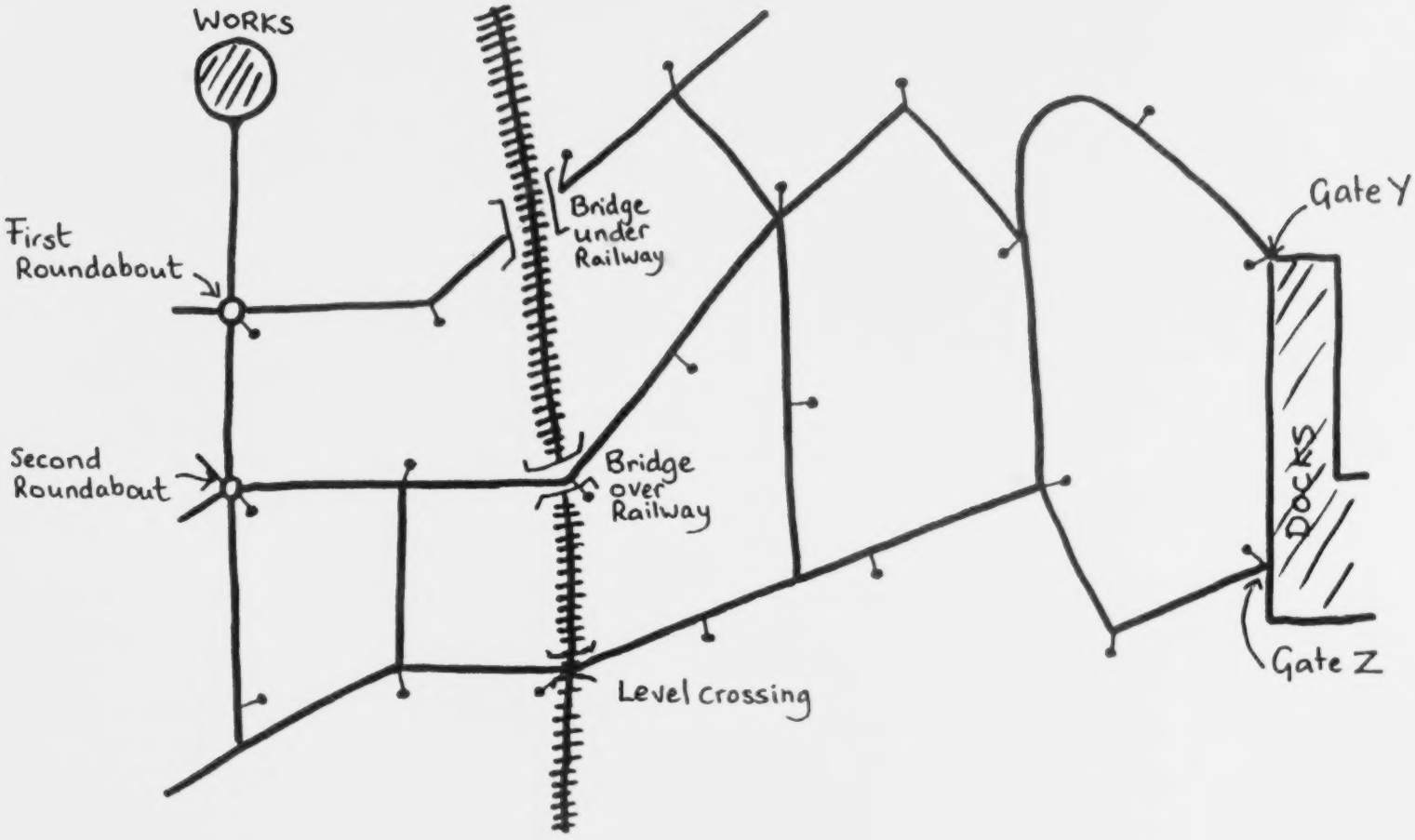
    If not, what prevented that from happening?

11. Could you have done the problem more quickly:

    On your own?

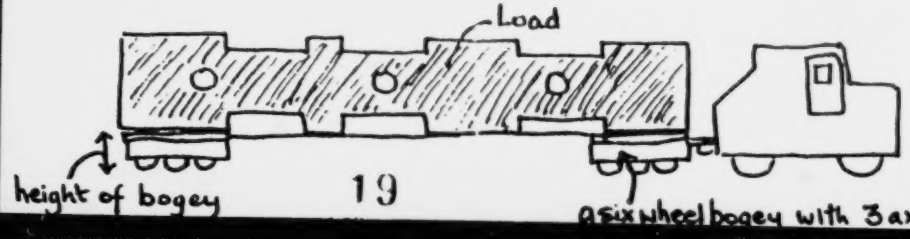
    In a different group?

12. What mathematics did you use to solve this problem?



Distance between ↑s = ½ mile

12



19

20

THE INDIVISIBLE LOAD

One of the problems facing an Engineering firm is the transportation of a large steel vessel from the firm's works to the nearby docks.

The major obstacle is a railway line. If you use the level crossing there is an additional insurance premium of £1000 to pay.

On the day the load is to be moved the crossing gates can only be kept open between the following times :-

0540 to 0600
0635 to 0655
0710 to 0730

You also know that with a load of this kind, many delays occur through having to manoeuvre around obstacles. For example, it will take 30 minutes to go "straight-on" at a roundabout, and 10 minutes to negotiate a 90 corner. It would take 20 minutes to negotiate the level crossing, because of the limited clearances.

Lighting up time on the day in question is from 21.30 to 04.35.

The attached map shows the roads which may be used. What will the total cost of insurance be ?

Which route should be taken ?

THE INDIVISIBLE LOAD

One of the problems facing an Engineering firm is the transportation of a large steel vessel from the firm's works to the nearby docks.

The vessel has the following dimensions :-

Length 87 feet, Diameter 16'7", Weight (including two bogies) 74 to

The vessel must be delivered to the Dock gates no later than 0745 hours. The major obstacle is the railway line, the bridge under this railway has headroom of 19'6" and the level crossing is very narrow.

Of the bogies available, Type A has the following dimensions.

No of axles	4
Height	2'4"
Max Speed	2 mph.

The attached map shows the roads which might be used.

What time should the load leave the works ?

THE INDIVISIBLE LOAD

One of the problems facing an Engineering firm is the transportation of a large steel vessel from the firm's works to the nearby docks.

The main obstacle is the busy main-line railway. The bridge over this railway is prohibited to vehicles with axle-loadings greater than 8 tons, and the level crossing has an effective width of only 17'0".

Of the bogies available, Type C has the following specifications.

No of axles	6
Height	4'3"
Max Speed	2 mph.

By law, a load of this kind cannot be moved during the hours of darkness.

The attached map shows the roads which might be used. Which bogie should be employed ?

THE INDIVISIBLE LOAD

One of the problems facing an Engineering firm is the transportation of a large steel vessel from the firm's works to the nearby docks.

The vessel is an absorbtion column, which must be delivered to the dock gate no later than 0745 hours. The major obstacle is a busy 4 track railway.
The bridge under the railway is very narrow, and 20 minutes should be allowed for the load to negotiate this bridge.

Of the bogies available, Type B has the following specifications:-

No of axles	4
Height	3' 6"
Max. Speed	5 mph

Because the roads are norrow, there is no bogey capable of negotiating a corner of less than 90 .
An insurance premium of £10 for every minute that this load is in transit will have to be paid.

The attached map shows the roads which might be used.
What time will the load actually arrive at the dock gates ?

HOW MANY LINES OF SYMMETRY?

Maureen Schreiber

Exploring geoboards and rubberbands with second grade students was more exciting than the problems they tried solving and worth every minute of the three forty-five minute sessions devoted to it. The overhead transparent geoboard was a manipulative these youngsters were ready to attempt to use as a creative medium in addition to the geoboards and colored rubberbands I had originally given them. Art and math are wonderful together!

Each of the 24 children had their own geoboard and colored rubberbands to explore. After extending the needed ten minutes of free exploration of the geoboards in the first session, the class reproduced on their geoboards the designs A-E from the distributed ditto. Each student was asked to divide the polygons into two equal parts. When they were asked for a definition of symmetry, one girl replied it must have a line which divides the shape in half. We allowed the definition for the first example, but when we found two lines of symmetry for the second polygon a new definition was needed. By the third polygon one student wanted the third polygon, a rectangle, to have three lines of symmetry including the diagonal of this rectangle. This brought up another rule for the definition of symmetry -- the mirror image. Students then made their own polygons on geoboards to find symmetrical lines. Each shape was recorded on dot paper. In the future I will use geoboard grid paper for young students instead of just plain dot paper. Many recordings from the students were not accurate using dot paper. Some sides of shapes were ten to fifteen dots on one side.

The second session was very popular. As a class we found how many lines of symmetry are theoretically found in various fruit before we sliced and shared them. Students then looked at various pattern block designs to determine how many lines of symmetry there were. The students had difficulty counting the lines. Once a design was divided, I had to erase the division so they could search for further symmetrical lines. Small mirrors were used to help those who still had difficulty or those who wanted to further extend the principle. The second session was finished by defining symmetry in another manner by folding crayon pictures in half and rubbing the image to complete a symmetrical picture.

The third session involved investigation of the restrictions of symmetrical designs by exploring how many different three, four, and five sided figures are symmetrical. Most of the time was spent on three sided figures. Students at this age still have difficulty with the concept of triangles except in the shape of an equilateral triangle. The assignment began with the student making a three sided polygon on his geoboard. Are all the sides the same length? How many sides have the same length. How many lines of symmetry can you find? Continue making triangles until the possible triangles of different sides, etc. are made. This may be a class problem all by itself. Those students who accomplished this task went on to four sided figures.

The following list of questions were given to the students to answer.

Do any/all three-sided figures have three lines of symmetry?

Will all equilateral triangles have three lines of symmetry?

Will all isosceles triangles have one line of symmetry?

Does any triangle not have any line of symmetry?

Make a four-sided figure that has four equal sides. Find the lines of symmetry.

Make a four-sided figure that has two pairs of 2 equal sides.

Make another one with a different shape. Find the lines of symmetry.

Make a four-sided figure that has only two equal sides. Find the lines of symmetry.

Can a four-sided figure have only three sides the same length?

Do any four-sided figures have four lines of symmetry?

Is there more than one four-sided shape that has four lines of symmetry?

How many four-sided shapes can you make that have only two equal sides? Do they all have only one line of symmetry?

Find a five-sided figure that has five lines of symmetry.

Find a six-sided figure that has six lines of symmetry.

Is there a special rule in finding the same number of lines of symmetry as the number of sides in the shape?

The objectives about symmetry which are required for the second graders were met easily after these lessons and I have no doubt that they will be remembered. The students were anxious to continue, a good sign in active learning.

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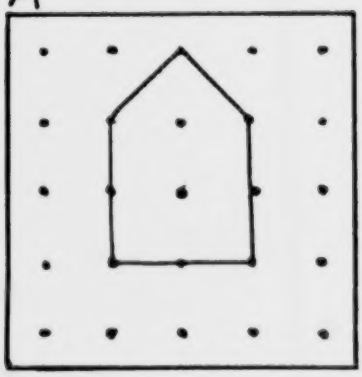

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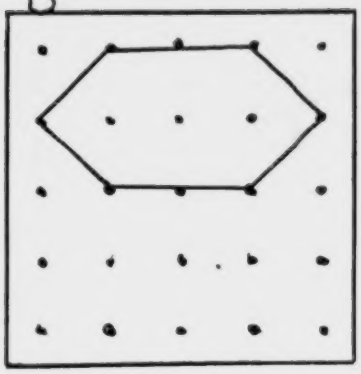

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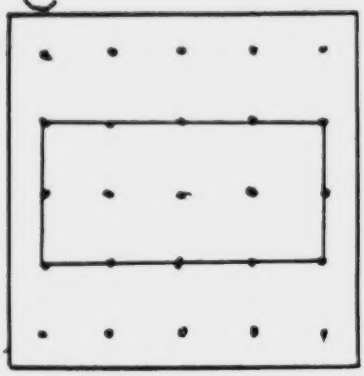

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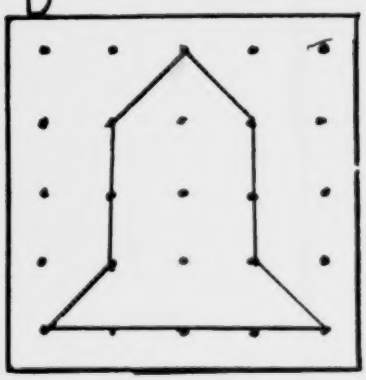

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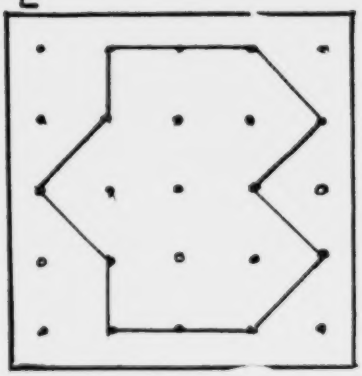

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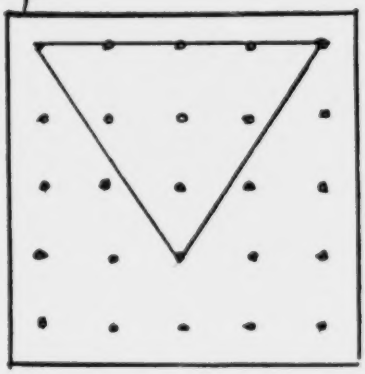

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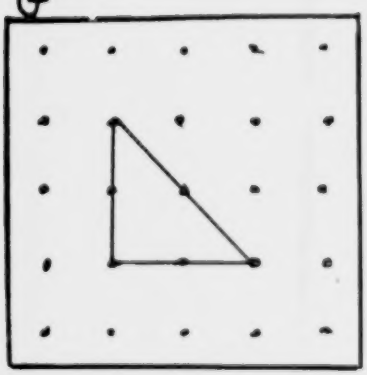

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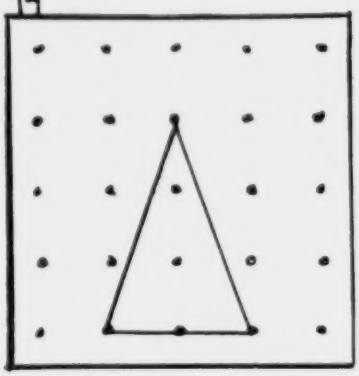

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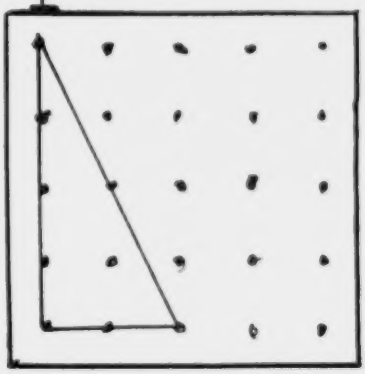


28

THIRD GRADERS AND MULTIPLICATION

Oleta Kartmann

Third graders practice interminably on multiplication facts and problems. But do they understand the process of multiplication? Dr. Thomas C. O'Brien, in a recent class session, startled us by saying that research indicates that 75% of fourth graders can not generate word problems for real life situations of 3 X 6 = 18. Surely "my" third graders would be able to do that! They have had many examples and much discussion about what multiplication is. I decided to investigate their proficiency in writing word problems for multiplication.

In preparation for this exercise the class reviewed a page of word problems from the basal text and discussed how the problems were stated. Several students volunteered appropriate examples. They were ready to begin. I wrote some multiplication facts on the chalkboard, then included some 2 - digit by 1 - digit problems as well. Each of these problems was assigned to a group of two to four students with the request that each student should write two word problems for his number fact or problem.

They started working. Difficulties arose. Immediately several children complained that they didn't know what to do. Again, using the example 3 X 6 = 18, I wrote a word problem on the chalkboard, explained the information that was needed, and how the question could be stated. Then I erased the example and no more help was given. A few individuals who asked for help after this were instructed to "go back to your seat and think it over." After several minutes the students appeared satisfied with their work and gave me their papers.

I told the children we would use their problems as practice sheets for the next day. At first I intended to eliminate those problems which were inconsistent with our model multiplication word problem. However, I decided a greater learning experience would result if I reproduced one of each set of problems written, just as the student had constructed it, correct or not. This resulted in three worksheets with twenty-six problems in all.

Examples of the various types of problems are as follows:

(1) 9 airplanes. 2 flights per day. How many flights?

(2) John had 9 shirts. 2 girls messed up two of them. How many in all?

(3) Kelli had 46 bags of sugar cookies. 3 cookies in each. How many in all?

(4) There were 3 children in the room. 46 more came in. How many in all?

(5) 46 bananas. 3 in each package. How many in all?

(6) Dan sucked on a lemon for 46 hours. He passed out 3 hours. (WHAT IS THE PROBLEM?)

(7) I have 7 cats. 3 whiskers fell out of each cat. How many whiskers fell out?

These examples show the variety of thought patterns among the children, all of whom were instructed to do the same task -- write multiplication problems. Seventeen of them constructed their problems correctly; nine did not.

The next day when the third graders began trying to solve their word problems, confusion reigned.

"This isn't multiplication!"
"How do you do this one?"
"This one doesn't ask anything!"

I instructed the class to solve each problem according to the operation that was needed, or to write an explanation as to why this was not possible.

After I checked their papers, I grouped their errors into three

categories:

Errors of Omission

8 students omitted 1 answer each
4 students omitted 2 answers each

Errors of Computation

3 students had 0 errors
5 students had 1 error each
7 students had 2 errors each
4 students had 3 errors each
3 students had 4 errors each
1 student had 5 errors
1 student had 6 errors
1 student had 10 errors

Errors of Interpretation

1 student had 1 error
3 students had 4 errors each
2 students had 5 errors each
5 students had 6 errors each
8 students had 7 errors each
7 students had 8 errors each
1 student had 9 errors.

The total errors in each group are: 16, 64, 174. Misinterpretation of the word problems caused almost three times as many errors as poor computation.

These results make it obvious that even though 2/3 of the class could write multiplication word problems correctly after having reviewed model problems, only one of the 27 students understood arithmetic operations well

enough to distinguish between the various operations represented by the children's word problems. Problems which were actually addition, subtraction, or division were often computed as multiplication. Few children offered a written explanation of why some of the problems were incomplete.

Why do third graders not understand word problems for multiplication? An incident that might provide a clue to the "why not" is the reaction of Jason to this problem:

Three children are in a room. 46 more came in. How many in all?

Jason multiplied to find the answer. When questioned, he argued that he had done the problem correctly. It HAD to be right! I drew a rectangle on the chalkboard and drew three circles to represent 3 children inside it. Then I drew 46 circles to represent the "46 more" and enlarged the rectangle so that it contained all the circles. Jason watched intently. "Now I see!" he said. "It's addition!" He was the author of this word problem and he thought he knew what 46 X 3 meant. Obviously, the 3 to him meant 3 physical objects, not the number of sets of 46.

My conclusion from Jason's example is that although third graders can calculate multiplication by using a mechanical procedure, what many of them visualize is, a set of 46 and a set of 3. It appears that students of this age level have difficulty understanding the abstract nature of numerical symbols, and think of them as representations of concrete objects.

CATEGORIZATION WITH AN 18 MONTH OLD

Lisa M. Laing

In my reading of **THE EARLY GROWTH OF LOGIC IN THE CHILD**, by Barbel Inhelder and Jean Piaget, I came across the statement that children between the ages of 6-8 months and 18-24 months show characteristics of classification although their language is not developed. As the aunt of an inquisitive 18 month old girl, I found this information to be "right on target." Heather, my niece, is at that wonderful age where everything is new and exciting; no object is too dull to investigate.

While watching her a few weeks ago I noticed that she was indeed using classification. Once I noticed this, it was everywhere! Heather loves to investigate the kitchen, systematically emptying the cabinets and the drawers. She first went to the cereal cabinet, took out three boxes, one-by-one, and sat them on the floor. Next she opened the cabinet with pots and pans inside. She giggled when she saw the metal shine in the light. She carried out four lids and two pie pans. Finally, she found two wooden spoons that were on the table. She sat on the floor with these objects all around her and got right to work. She began picking up the objects at random, making faces in the pie pans and making stirring motions with wooden spoons. I soon began to notice that as she was examining the objects, she was sorting them. She put all of the pie pans and lids together, and the cereal boxes and wooden spoons together. While she was amazing her aunt, Heather jumped up, ran into another room and returned with a book in her hands. She put the book on one of the cereal boxes. It appeared to m e that she was classifying according to shiny versus dull surfaces. I wondered if she knew how astonished I was, because, after this last step, she looked at me and ran to give me a hug. I also wondered how often she did this type of activity, and I soon found out. Within the next hour and a half, before Heather took her afternoon nap, I counted seven more instances where she appeared to be categorizing objects. These categorizations ranged from objects that belonged in her school desk (chalk, magnets and alphabet letters) to her parents shoes, which she sorted into pairs. After studying the hard work that this 18 month old performed, it is no wonder she needed a nap!

ATTRIBUTE BLOCKS
Connie Williamson

I have always had a interest in watching children interact with their peers and siblings. Dr. O'Brien's class encouraged me to begin "child-watching" which entails a great deal of thought, discovery and experience. I have been using one of my favorite activities, attribute blocks, with EMH students who were successful and motivated with the following game.

The materials can be the real attribute blocks or a set of fake blocks made of paper. The task was put the blocks into groups and label them. The participants were EMH teenagers.

The students first put the blocks into four groups based on color: red, green, yellow, and blue. I expected them to at least be able to separate the colors. I praised their accomplishment and asked for another way. One child did a grouping of big and small while others did a shape grouping of squares, diamonds, circles, and triangles. I then saw a boy with a group and immediately thought he had misunderstood the directions.

He had a group of a small yellow circle, a large red square, a small green triangle and a large blue diamond. I asked Michael what his label was for his group. I thought he'd say he didn't have one and would regroup. Instead he looked at me as though I were EMH and said "these aren't the same color or shape". I was stunned. Because of their label of EMH, I had overlooked the possibility that these kids would see the dissimilarity of the blocks.

After seeing how Mike had stumped the teacher, they all began their not alike groupings. I had never seen such pure excitement in problem solving and discovery thinking. These children were problem solving and through their discoveries they were learning. It was pure learning. I gave no clues or grades. I just monitored the level of excitement. Thanks to Tom O'Brien and Mary Steffen, I have become a fascinated child watcher.

AN ARGUMENT FOR AUTONOMY
David Hoffman

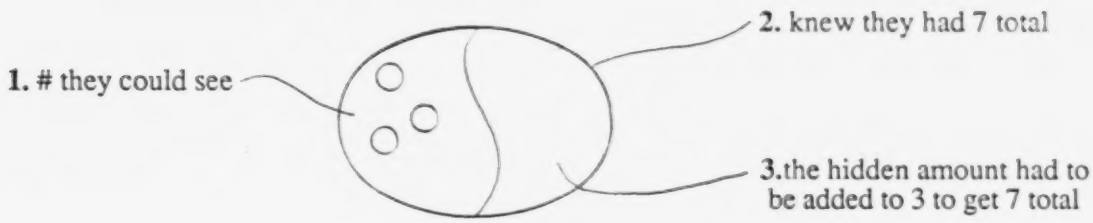
I performed the hidden chip task with 3 kids, all three were 7 years old.

The task involved showing 7 chips on a table, covering a certain amount with my hand, and then asking how many were under my hand. Two of the kids solved the problem in the same way (at least outwardly). They counted the showing chips by pointing to them one at a time and then used their fingers to add on to that amount until they got to the number of chips they knew we had started with.

I.E. (four chips covered):

They would then look at the number of fingers up and say "4".

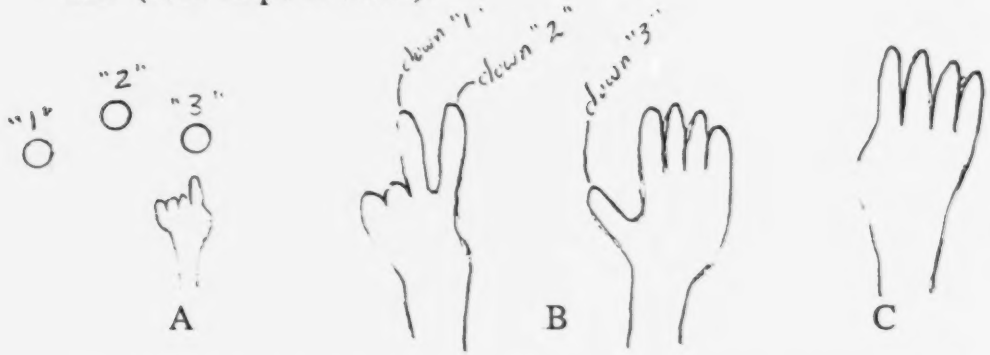
This could be represented by:



The other child I worked with solved the problem differently. We had 7 total, I covered 4. He counted the 3 that were showing by pointing to them,

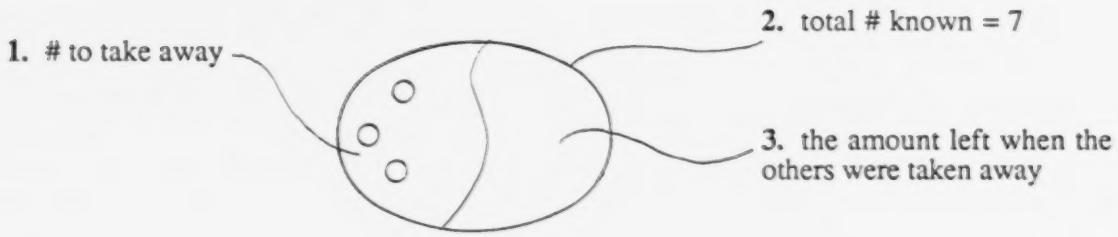
and then held up 7 fingers. He counted down 3 fingers and had 4 left up, responding "4".

I.E. (four chips covered):



He then looked at the number of fingers left up and responded "4".

This might be represented by:



I could have tried to make this child perform the task in the same way as the other child, but for what reason? The desired result was achieved both ways.

What does this have to do with autonomy?

I couldn't help thinking, what made these kids different? They come from different backgrounds, have different genes, different interests, different diets, the list could go on indefinitely. Yet, we as teachers expect them to perform in situations exactly the same way. We expect them to solve

problems exactly the same right way.

Although kids pass through the same Piagetian stages of development, the ages at which they do it vary greatly.

Given all these facts, why don't we provide an environment where the individual need is stressed over the need of the masses? We may have a classroom full of 9-year-olds, but their difference in developmental stages may make them seem like 5 to 14 year olds.

In an autonomous classroom where the environment allows for individual work at a pace directed both by the teacher and the student's feelings of responsibility and readiness, motivation is natural. A child can call on his constructs and apply them to solve a problem, maybe not his friends way, maybe not the teachers' way, but his way--He owns the solution.

Is the issue the method he used, or whether or not he was successful in solving the problem?

We may be successful at getting kids to recite back the math facts in the type of environment we have in our classroom, but are we allowing the individual to use his past experiences in application to problem-solving? An autonomous environment does allow the individual to apply what he knows to solve new situations given him in the classroom using a method he may invent himself and calling on higher level thinking.

Isn't that what school should be about?

PATTERNS FOR COUNTING ON
Linda Carroll

I gave each of my 22 second graders a piece of 2" graph paper. There were eleven squares across, eight squares down.

I asked them to number consecutively the first row only.

Teacher: What is the last number at the end of the row?

Children: Eleven.

Teacher: Try not to count any further. From looking at the first row, what do you think the last column will look like over here when you get finished?
(I drew the last column on the board, putting an eleven in the top box.)

(Several looked puzzled and one said, I can't tell." Then silence.)

Josh: I know! It will be 22, 33, 55, 66, 77, 88, 99, (Almost shouting.)

Teacher: How do you know?

Josh: Well, 10 more would be 32, plus 1 is 33.

Teacher: So the next row would be ___?

Josh and a few others who had caught on: 44!

Teacher: And the next one?

Children: 55!

Readers, I submit to you that this is thinking at its best. Can we stretch ourselves as teachers to challenge children to think in a meaningful, exciting way?

SOME THOUGHTS ABOUT TREASURE-KEEPING

Thomas C. O'Brien
Teachers' Center Project
Southern Illinois University
Edwardsville, IL 62026

A thought experiment: Imagine that a friend has entrusted to you a Rolls-Royce. Or, if it's more comfortable, imagine that a neighbor has turned over to you a very rare valuable tropical plant for temporary safekeeping.

What would you do?

At the outset, you would probably be frightened by the responsibility wished upon you. Then you'd probably try to find out all you could about the care of Rolls-Royces (or rare plants of the species entrusted to you). You'd hungrily read what knowledgeable people had to say. Beyond that, you'd contact experienced friends and colleagues to ask their advice. You'd constantly check that the recommendations did, in fact, work. And you'd probably keep an eye on the treasure entrusted to you to learn about issues not covered by available expertise. That is, in new situations you'd gently try new tactics and you'd assess what you tried to see that it acted on behalf of the well-being of your treasure.

What if you were an educator and the treasure entrusted to you were a child's mind? Probably the same. You'd find out as much as was known about children and their thinking and try it out, constantly monitoring that the advice had good effect, and you'd keep an eye out for issues not mentioned by the experts so that you might become, in a small sense, an expert yourself.

What American educators did in addressing the issue, was to adopt a "solution" which has dominated education for more than sixty years and shows no sign of waning. What they did was to adopt policies and procedures directly from the factories and assembly lines of the rapidly industrializing America of 1910-1920. In a history of the underpinnings of American schooling, Callahan documents the domination of American education by factory-oriented thinking centered about three themes: 1) mass production practises, 2) cost-effectiveness, and 3) time-motion studies carried out by "efficiency experts," one of which showed that a worker, Schmidt, increased his daily haulage of pig iron from 12 1/2 tons to 48 tons when his salary was raised from $1.15 to $1.85 per day. (Note 1)

Given such underpinnings, American education commonly sees children as raw material to be stamped into shape, empty urns into which "stuff" called knowledge is poured. And it is the quantity of stored "stuff", rather than the approach itself, that is assessed.

In this essay I suggest an alternative to the factory-based, mechanistic underpinnings so commonplace in American education. The points raised here are still being sorted out and there may well be entire areas missing. Readers are invited to fill in, revise, and extend what's here in the interest of constructing their own guidebook to the treasure, a child 's mind, that is entrusted to them.

The observations in this essay are Piagetian in the sense that they rely heavily on 60+ years of research on the origin and development of knowledge in children by Piaget and his collaborators around the world (Note 2.) and they draw on my experience with my own three children (to whom I owe an immense debt) and on my experience with thousands of children over 25 years as a teacher and teacher-of-teachers.

To start, let me give you a quiz. Name the capitals of the following states:

1. New York
2. California
3. Massachusetts
4. Texas
5. Indiana
6. New Hampshire
7. Delaware
8. North Dakota
9. Alaska
10. Mississippi

Now try these:

11. Canada
12. Newfoundland
13. Prince Edward I.
14. Quebec
15. Manitoba
16. Ghana
17. Chad
18. Papua New Guinea
19. India
20. Zimbabwe

How do you feel? Having administered this test to thousands of teachers, I can guess that you are saying 1) "Wow, am I stupid!" or 2) "Who cares? This is

foolish. Zimbabwe's capital is not a major concern of mine right now."

A great many American schoolchildren feel stupid as the result of present school policies. A great many schoolchildren say "Who cares?" The dropout rate for the City of New York public schools is variously estimated at 36-45 per cent. The national average is 29 per cent. (See Note 3.) (The figures refer to those who have physically dropped out. How many more are physically in school but spiritually elsewhere?) If your physician had a client-mortality rate of 29 per cent some serious charges would be levelled, I suspect.

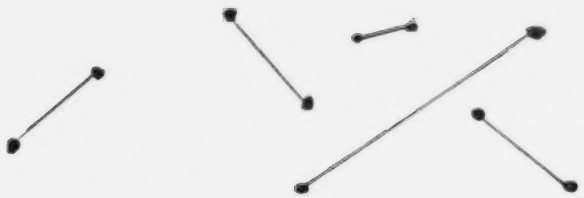
Now, please, another test. Write all you can about San Francisco.

Here are two responses to the question:

> I Left My Heart in San Francisco. San Francisco is the second most beautiful city in the world, with brisk air, perky self-confident people, stunning sights, superb restaurants and fabulous shopping. There's not enough time to do it now, but I could tell you about La Quiche Restaurant, the Beresford Hotel (old-fashioned, British flavor, known from Saturday Review in the 1960's), almost vertical hills for driving and walking excitement, theater and dance and opera. The streets around Union Square provide more visual interest than most whole cities in the rest of the US.

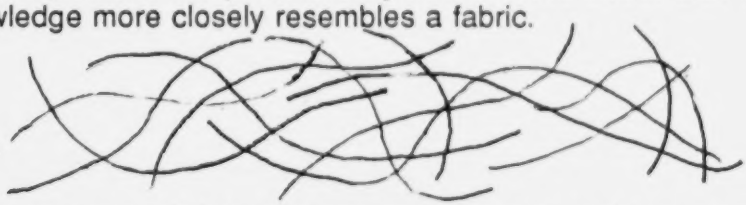
> Alcatraz (I wonder what is is now?). Many gay males. Tolerance of alternate life styles. One should visit the Exploratium! And the waterfront, with its shops (There's an entire shop given to chocolate.) and people-watching and crowds. The second biggest city in California. Earthquakes. Earthquake and fire (1906?) Cable cars. Asians and Chinatown and diminishing Italian neighborhoods. City of St. Francis. Giants. Steep streets. Golden Gate Bridge. Fog. Fog. Fog.

THE NATURE OF THE TREASURE. What is this treasure called mind? It KNOWS. What does it mean, "to know"? The first point to make is that knowledge is not a bunch of rote associations -- New York (Albany), California (Sacramento), ..., Zimbabwe (Harare).



But much of American education consists of teachers' transmission of associations -- or chains of associations -- to children. Dead Mice Smell Bad is a major topic in education in many sixth grade "math" curricula. What? Yes, math! Divide, Multiply, Subtract, Bring down. That's how you do division. What an appalling view of knowledge. Of math.

Rather, as shows up universally in the "write about San Francisco" task, one's knowledge more closely resembles a fabric.



It's a network of information, images, relationships, errors, hypotheses, anticipations, inferences, inconsistencies, gaps, feelings, hunches, rules, generalizations, and so forth. If you think of neuronal networks when you look at the picture above, you're on tne right track.

ORIGINS AND ENDS. The two sorts of knowledge differ very much in their origins. The first type of knowledge is obtained by copying. Children copy it from the textbook or the teacher or from notes they tape to their wrist.

The second type of knowledge is constructed through encounters with the world. You prowl the streets of San Francisco. You go to see the Giants at Candlestick Park. You get lost; look up old friends, park your car on a scare-making hill. You discover a fabulous restaurant or buy a raincoat or sail under the Golden Gate Bridge.

The two sorts of knowledge differ also in their accessibility. One can train a parrot -- or a child -- to associate the nonsense syllables "2+3" with the syllable "5." But there's no chance that the parrot (or the child, if that's all he knows) will

say "five" when shown side-by-side piles of 2 and 3 pennies.

On the other hand, one's mental fabric is richly accessible. Suppose I say to you the word "rent." You think of the payments you make for your home. You think of the wonderful bicycle trip last summer in Holland. You think that the British usually use the word "hire." You think of legal contracts and lawyers and landlords. You think of the Bible, where garment was rent (in another sense of the word).

And suppose I asked one of the writers of a "San Francisco" story to elaborate on an aspect of his story -- the San Francisco Giants or the Golden Gate Bridge or "fine dining", for example. There's no doubt, given that the original story included these elements, that he could do so. One aspect of a fabric is that it contains mini-fabrics.

Another distinction is that rote connections are usually seen as ends in themselves. When was George Washington born? 1732. That's usually the end of the issue. Did George Washington speak with a British accent? What sort of money did he use? What schooling did kids have? Did young George revere the Crown? What games did kids play? What would be familiar to young George if ne were to live in your town? What would he find amusing? Surprising? Offensive?

But fabrics are both beginning and end. One of the most important characteristics of mental fabrics is that they generate new knowledge. Ask a five year-old to show you with fingers 2+2. Then ask her to show 2+3. For many children at this age, 2+3 is not a new atomistic connection separate from 2+2. It's derivable from 2+2. It's 1 more that 2+2.

Knowledge is not accretional -- formed brick by brick as is the case with assembly lines. Rather, it evolves.

TENDENCIES. The fabric of knowledge has several important tendencies. It tends toward coherence, stability, generalizability, and economy. These tendencies enable children to organize the blooming, buzzing confusion that surrounds them.

One other tendency may be the most important. The fabric tends to quest.

That is, it is not satisfied to have reached equilibrium; having done so it searches out or invents ever higher challenges to conquer. A child first approaches a playground slide very gently. She holds onto the sides and drags her feet on the first go. Ooops, come back a month later and the child is climbing up the slide from the "wrong" end. And perhaps she then slides down full speed, maybe even backwards.

A central problem in life is to organize reality. (Think of the organizing you do when you visit a city new to you. Your fabric is sparse at first. It relates the Interstate to the hotel to the lovely restaurant you visited last night. Stick around another day and your fabric is likely to be intrinsically different and more complex.) What happens in American schools, from grade K through graduate school, all too often, is that teachers organize things for children and turn over that organization, full-blown, to children to remember until test time. The tragic consequence is that children facing a new problem often say, "I can't do that. The teacher hasn't taught me yet." And teachers facing a new class in September commonly say, "These kids were taught last year but they didn't STAY taught."

UNDERLYING ACTIONS. There are at least four very important actions that the mental fabric conducts in its encounters with the world: construction, coordination, elaboration, and differentiation.

One coordinates within and among one's fabrics and between one's fabrics and external reality. Physical coordination is so easily apparent on the playground, but we give little attention to mental coordination. Yet even the newborn infant is coordinating space and time and cause and effect and memory as she wriggles in her crib to make an overhead mobile move.

Elaboration is one's alteration of fabric to accommodate new situations. I drive a Toyota at home and when on travel I rent a Ford, I have to elaborate my Toyota network to accommodate to Fords. Elaboration is quite different from transfer, I think, and it is much more common. Transfer, as it is widely understood in education, refers to the application of a fixed chain of actions to new situations. When they are part of a fabric, actions are not fixed. One's mental fabric is constantly under construction and susceptible to accommodation.

Differentiation is the reverse of elaboration. At one time a child may learn that those horned things in the field are "moo moos." Later he sees some goats

and calls them "moo moos." He must differentiate his class of horned animals into moo moos and goats (and at a later time, deer and elk and unicorns).

Knowledge is a personal construction. Your "San Francisco" is different from mine (with lots in common). Further, the constructions of children are intrinsically different from those of adults. Their fabric is not merely smaller than ours (with some exceptions: I learned from a five-year-old that Henry Hudson died after a shipboard mutiny), it is made of different stuff.

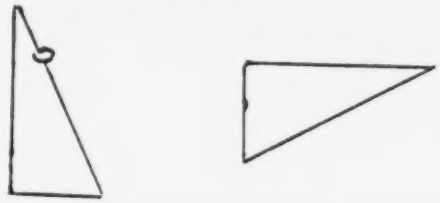
NECESSITY. As one sorts out the world's buzzing confusion, logical necessity -- the notion that such-and-such must be (or cannot be) so is a life-long construction and a major bit of glue which holds one's fabric together and helps to extend it. (Logical necessity, please note, is different from mere convention. It must be so that Americans drive on the right side of the road and that certain forms be filled out in triplicate. Conventions, by their very nature, are arbitrary, not logically necessary.)

Show five chips to a child and ask him to count them. Then ask him to close his eyes while you cover some of the chips. Then, eyes open now, the child is asked to tell you how many chips are hidden. An eight year old will tell you, three showing, that two MUST be hidden. The child has coordinated the three classes -- the chips showing, the original total, and the chips hidden, to infer with necessity. Try the task with a four-year-old. NO such thing, quite likely!

SPECIFIC ACTIONS. There are a number of important actions which hold one's mental fabric together and which extend it. Probably the most primitive such action is comparing things to check whether they are the same or different. (The Genevan psychologist Hermine Sinclair tells of childwatching in a creche in Paris. Two very young infants touch hands spontaneously. Their hands mismatch. Slowly and gropingly the children rearrange their hands until finally their palms match perfectly. Both infants light up with broad smiles!) This "same vs. different" testing, begun among newborns ("Is this the person who brought me nutrition this morning?") flourishes throughout life: adults run analyses of covariance to assess the probability that this or that sample was drawn from the same population.

The notion of "same" versus "different" is intimately involved in the various conservations (assertions of invariance over transformation) which one

constructs, ranging from "Does the marble still exist after it rolls under the couch?" through to "Place the penny in the same place on the figure at the right," and beyond. (Note 4.)



Other major acts of "glue" which hold one's fabric together and extend it and in general organize the world's buzzing confusion are classifying, ordering, logically multiplying, inferring, reasoning propositionally, reasoning causally, thinking combinatorially, reasoning probabilistically and proportionally, keeping all but one variable constant to assess its effect, and so forth. (Within classification, as with the other actions, is a host of important aspects such as hierarchical classification, nesting, transitivity, and so forth.)

And then there are more global actions such as hypothesis-making, hypothesis-testing and theory building (At an early age American children build the sensible theory that one pluralizes a noun by adding an "s"; thus, "foot/foots, tooth/tooths, etc.) Sad to say, hypothesis making and testing and the immensely powerful notions of pattern-searching, pattern-devising, and pattern-testing get little attention in American education in favor of "basics" such as California (Sacramento), Zimbabwe (Harare), 9*3 (27), and "Dead Mice Smell Bad."

RELATIONS. All the actions above are concerned with action on and internal coordination of the entities of one's network. To infer that A must be heavier than C because A is heavier than B and B is heavier than C is knowledge of this sort. But one can also gain knowledge by acting directly on observable reality. For example, one can tell by handling objects that this object is heavier than that. (Note 5.)

A major part of one's fabric is gained from acting upon observable reality. "Same as" is a major relation (with, as in the infants' example above, certain aspects of the situation such as skin color and hand size suppressed). But there are many other local linkages -- father of, west of, lives near, above, smaller/earlier/skinnier/less/younger than, wed to, meets at, which

connect/relate/separate things.

Within a domain there are context-specific relationships which are not very useful when one moves to a new context. For example, the Johnsons are richer than the Smiths. The next term in the sequence 34, 28, 23, 14 is Astor Place. (These are subway stops in Manhattan.) Channel 2 is ABC in St. Louis.

INFORMATION. At last we get to information, facts, and descriptions, the stuff of most school curricula, much of which is transmitted, fully-formed, by teachers. Lincoln was the sixteenth president. Columbus discovered America. Such and such is the procedure for square root. Harare is the capital of Zimbabwe. George Washington was born in 1732.

Despite our best efforts in favor of neat, clean associations, even information, facts, descriptions, and procedures are constructions. Search your own fabric for a meaning to the "facts" above now that you've read this far. You probably have a fabric for all of them and that fabric may well have changed in the short time you've been reading this paper. (See Note 6.)

CONVENTIONS. Finally, there is information which is arbitrary but which, for the sake of communication, we agree to agree. ıron is "Fe". "+" means add. Certain sentences end in a dot. My first name is spelled T-h-o-m-a-s (but in Hungary it would be T-a-m-a-s). Scalene triangles have three acute angles. If two vowels do the walking, the first one does the talking (as in "poet" and "soup"?). Conventions can be learned by rote, it might seem, but even here fabrics begin to grow. (Note 7.)

CONTENT. This whole essay purports to be concerned with the education of children. What about content? Education is widely seen as concerned with content!!!

I propose that I have written about content from the beginning of this paper. Content is one's fabric, with its origins and ends, tendencies, underlying actions, necessities, specific actions, relations, information, and conventions, as described above. Your content, e.g., "San Francisco," is your personal construction, your network, with all the issues above (and probably more) having come into play.

I started this essay asking the reader to imagine having been entrusted with a treasure. Vastly more a treasure than a Rolls-Royce or a rare and valuable tropical plant is the mind of a child.

Yes, it's an awesome responsibility we have as teachers. It's one we cannot leave to assembly line policies and practises. Children's minds, as I hope I've made clear, are not inert iron waiting for the forge.

The next question, having got a glimpse of the issue, is how to care for, nourish, enrich the treasure entrusted us for so short a time. THAT's the task we teachers face, surrounded as we are by a wrong-handed view of children's minds and a barren understanding of what it really means "to know".

NOTES

1. Callahan, Raymond E., **Education and the Cult of Efficiency**. Chicago: University of Chicago Press, 1962.

2. The original writings of Piaget are difficult to read, not to say anything of their sheer bulk. For excellent introductions to Piaget and his work, the following readings are suggested:

Ginsburg, Herbert, and Opper, Sylvia, **Piaget's Theory of Intellectual Development**. Englewood Cliffs: Prentice-Hall, Inc., 1979.

Boden, Margaret A., **Jean Piaget**. New York: Viking Press, 1980.

Labinowicz, Ed., **The Piaget Primer. Thinking, Learning, Teaching**. Menlo Park: Addison-Wesley Publishing Company, 1980.

3. The 29 per cent national figure (Southern Growth Policies Board, U.S. Bureau of the Census, as reported in the November 23, 1986 issue of **The New York Times**, is the number of 1984 graduates as a percent of ninth grade enrollment four years earlier. The dropout rate in Louisiana, in the same report, was 43 per cent and that for Florida and Mississippi 38 per cent.

4. See Thomas, O., "Students' Understanding of Selected Transformation Geometry Concepts", cited in Dickson, Linda, Brown, Margaret, and Gibson,

Olwen, **Children Learning Mathematics: A Teacher's Guide to Recent Research**. London: Holt, Rinehart and Winston for the Schools Council, 1984. Only 40% of 6 year olds and 50% of 9 year olds accomplished the task shown.

5. See Piaget, J., "Phenocopy and the Development of Knowledge," in Gruber, Howard E. and Voneche, J. Jacques (eds.), **The Essential Piaget**. New York: Basic Books, 1977.

6. In fact, research by Brownell more than fifty years old (pointed out to me by Ed Labinowicz) showed much the same, though present school practises don't suggest that the research is known today. Third grade children whose math diet had consisted of two years of drill and practise on addition and subtraction "facts" produced, when interviewed, 37 per cent of the "facts" through home-made strategies. Says Labinowicz, "Unbeknownst to the teacher, who assumed the children were repeating the combinations, the children had taught themselves other ways of thinking in an attempt to make sense out of the combinations." Said Brownell in 1935, "...if arithmetic becomes meaningful, it is absurd to assign credit to drill. It is nearer the truth to say that if ... arithmetic becomes meaningful, it becomes so in spite of drill. "See Labinowicz, Ed, **Learning from Children.** Menlo Park: Addison-Wesley Publishing Company, 1985. The Brownell research is cited on p. 96. The entire Labinowicz book can be characterized as a research on children's home-made construction of elementary number knowledge.

7. As I wrote this, I thought about the convention "red traffic light = stop". In Brazil, I remember having been told, "red traffic light" means "Perhaps you might want to stop!"

The notion that even conventions, however arbitrary and atomistic, are supported and surrounded by fabric, is given credence by Ellen (5) who was asked to draw an "A". What could be more arbitrary than a letter of the alphabet? (Think of the various alphabets you have seen -- Anglo, German, Hebrew, Japanese, Arabic, Cyrillic.) Ellen, 5, had drawn up a fabric. "That's ('A') the TENT letter", she said, making a tent with two fingers.

For rich information on the issue, see Labinowicz, Ed, **Learning From Children**. Menlo Park: Addison-Wesley Publishing Company, 1985, and Ginsburg, Herbert, **Children's Arithmetic. How They Learn It and How You Teach It**. Austin: Pro-Ed, 1983.

A GOOD LEARNING ENVIRONMENT
Shirley Gasparich

This quarter Dr. O'Brien has challenged my classmates and me to describe the various aspects of a good learning environment. I recently read a profound analogy Erma Bombeck made between raising kids and flying a kite. I suggest that it also simply and very eloquently illustrates all the conditions of a good learning situation as well, where the teacher knows the child and provides him with just the right amount of mismatch -- intellectual/tension tautness -- for the student to stretch his construction to a higher level of thought.

According to Erma, both kids and kites are destined to soar free and independent, but the conditions have to be just right or all will fail. If the kite string is given too much slack, (freedom - not enough intellectual tension) the kite will come crashing back to earth. If the string is too taut, (too much mismatch or a stifling learning environment) the string snaps, and the kite is lost forever. If the person holding the kite string, keeps it taut, not too tight and not too loose, and if he constantly releases the string, allowing the kite (child) to catch the wind as it will, choosing its own direction, yet providing the firm guidance it requires, the kite (child) will soar free and independent.

This is not an easy goal to accomplish (especially with 30 in a classroom), but certainly a beautiful image and a wonderous feeling when it happens.

KIDS AS KITES
by Erma Bombeck

You spend a lifetime trying to get them off the ground. Finally they are airborne, but they need more string and you keep letting it out and with each twist of the ball of twine, there is a sadness that goes with the joy because the kite becomes distant and somehow you know that it won't be long before that beautiful creature will snap the lifeline that bound you together and soar as it was meant to soar.....free and alone. Only then do you know that you did your job.

When we fly kites, we must stop and repair them, untangle them from trees and wires, add a little more string, check the direction of the wind, all the while letting them fly a little higher, a little more bravely until they're on thei r own.

PARDON MY MISTAKE, PLEASE

Linda Kahley

Children make many mistakes during the course of a school day. I have always considered these mistakes to be a sign of what they do not know; what they need to have retaught; or a sign of carelessness. I have made a mistake in this thinking and have learned to reevaluate my thinking on children's errors.

Through the readings I have done for Dr. O'Brien, I have discovered that my construction for errors was too simplistic. I had not given the students credit for what they do know, nor had I seen their mistakes as the key to their thought processes. This is also the key to my new construction: analysis of the errors children make will tell me more about their knowledge and their thought process than will their correct responses.

As Herbert Ginsberg states in **CHILDREN'S ARITHMETIC: HOW THEY LEARN IT AND HOW YOU TEACH IT**, each and every child has hidden competencies which are often undetected. If the child's papers are marked only with an eye to tallying the correct/incorrect answers, those competencies may never be discovered. It is important, therefore, to analyze the mistakes the child makes in order to see where those competencies are. Of course, this takes more time than using the teacher's manual with the printed answers to grade papers , but it will tell more about the capabilities of the child. In all likelihood the child will have constructed a method of problem solving for himself and he will consistently employ that method.

In other words, the child formulates a method of problem solving based upon the knowledge he already has. Sometimes this is successful, because some solutions are based upon prior skills. At other times this is not successful, because the techniques one uses in one problem are not appropriate in another problem. Math can be extremely frustrating to a child, because the rules are constantly shifting. A child is taught that "5" means a quantity of items (there are five apples on the table). After a child becomes assured of the quantity of "5", he is introduced to the concept of place value. Here, "5" means something

totally different depending upon where it is placed (5, meaning five units, 50, meaning fifty, and 500 meaning five hundred, etc.) the child, in an attempt to make sense out of this instability, may write 502 for the number 52. Now, this appears to be a completely unacceptable response, and in terms of place value it is, but upon analysis of the mistake, it is apparent that the child is recording the quantity he has counted; 50 + 2. We can see that the child is thinking, I have 50 and 2 more, therefore I write this as 502. The child has constructed a problem solving approach for himself, it is incorrect, but it tells us how he is thinking. Similar mistakes are found in a child's recording of 20016 to denote 216, or 10010 to mean 110.

Another example of a child's construction is demonstrated by a second grade boy's responses as he was learning how to multiply. The first problem on the page was 3 X 2 =___. He looked at the pictures of the sets of objects printed by the problem, and counted 6 objects. Therefore he figured the answer to be 6. Then, he began to build his construction. He knew that 3 + 2 = 5, and he had just counted that 3 X 2 = 6, so he constructed an approach to multiplication that the answer will always be one more than the sum of the two numbers. So, his answer to 3 X 4 = ___ would be 8. This is obviously an incorrect construction, but this is how he used his previous knowledge about addition to devise his approach to multiplication.

These examples of problem solving are definitely mistakes, and they definitely show a need for further teaching and learning for that child. More importantly, however, they show the thinking and growing process of a child's mind, and for that reason they are extremely valuable and worthy of study.

COATS AND HATS - LINEAL DUPLICATIONS

Maureen Schreiber

Given three choices of paper hats and four choices of paper coats to be combined as hat and coat groups, a four year old was ready to begin the task of making all possible combinations. It began with the child taking one purple coat and a green hat. Next he went to the next color coat and the next color hat, yellow and pink. He followed with white coat and orange hat. With the last color coat, black, he chose a green hat. I was ready to assume he would begin again with the purple coat and pink hat, which he did. This was the last of his orderly method, so far as I could see. He did, however, try to make sure he was not duplicating any of the previous combinations.

Now I made a big error by moving his previous combinations to allow him more space in front of him. I had begun an arrangement of four across and began the second row of coat/hat combinations. Now he had difficulty looking for duplicates in the previous row. I also had to remind him to go to the previous row and check. I was sure I had made the task impossible for him to complete; he now would continue forever since he did not see his repeated combinations. This is a child that I know can match items by playing a 4 X 4 grid picture bingo. However he could not transfer that task to this problem.

After another setting I allowed him to keep his combinations in a line and he was able to complete his project, finding all the combinations. His only difficulty was determining that he had completed the task. He would keep making combinations and checking to find that he already had that combination. He would use a purple coat and try to find a hat to combine with it to make a new combination. After he tried all the hats for the purple coat, I persuaded him to push the purple coats away and consider that he was finished with them. He then continued this task with all the other colored coats until all the colors were gone.

Perhaps more problems could be solved if desks were larger for students to work lineally until they have developed the ability to work on a limited space. Consider also that printed worksheets use only rows or columns for students to look for answers. We force them to not work lineally.

SEVEN CUPS IN A ROW
Debbie Johnson

Using seven plastic cups I saw some of Piaget's theory in action. Our child watching task for the week was to take six to nine objects varying in size and ask a group of children to put them in order from largest to smallest without communicating to each other.

This is how the task was set up. I had two groups of five kindergarten children, eight green construction paper squares were put down and seven plastic cups of varying size were randomly placed on the squares leaving one square empty. The children were then told to place the cups in order from highest to lowest by moving one cup at a time. They were only to move a cup to an empty square and they could not talk to each other until they were sure the cups were in order.

I let the children begin. They studied the arrangement of the cups and the first go round was random moving of the cups. Then I noticed that they kept moving the largest cups next to the smallest cups, which was their interpretation of the instructions.

I later found that, for their age, this was a common practice. They were pairing large to small. I then removed three cups and three squares and gave more definite instructions. I also showed them three cups in a row and asked them if they could do that. They said yes and put the four cups in perfect

order. Then I asked them if they thought they could do it again with the seven cups. They said yes and began to try. In the beginning there seemed to be a powerplay as to which end the largest and smallest cup would be, but after a few of the similar size cups got to one end they let it stay. As they played I could tell that three out of the five girls knew what cups went where and the other two girls were stuck on pairing largest with smallest. The children who knew were frustrated and kept asking me if they could tell the others where to put the cups. In the end, despite the two who were still operating on pairing, the cups were put into order.

The second group of children were almost identical in their actions to the first group.

Through child watching, one sees the implications of cognitive development. I saw the difference between children who were pairing the cups, being able to concentrate on only one thing (large and small), and children who saw the series (more than just large and small). As one child put it "I know, we are filling in the squares."

CIRCLES

Maureen Schreiber

After participating in the Summer Workshop of the Teachers' Center Project, I presented a four year old with the problem of making all the possible combinations of two circles. Paper circles of two sizes were given to him. The method of making the various combinations was orderly, much to my surprise.

First, he placed only two large circles on the table, making all possible combinations. He had no difficulty deciding when this first task was complete. Next he chose two small circles and again made all possible combinations. It was noted that he looked at his first examples of large circles and duplicated the examples but used the small circles. Last, he took one small and one large circle. Again he duplicated the previous examples, but he did include the case where the smaller circle goes inside the larger circle.

He did not duplicate patterns by flipping or turning them. It is difficult to know if this was deliberate. Once he realized that he could put a small circle inside a large circle he did want to return to his small circles and place a small circle on top of another small circle. I prevented him from making this combination explaining to him that I had to see part of both circles for this project. When he finished his combinations, he did not look for any duplicates and decided he could not make any others. He was very self assured he had completed all that he could.

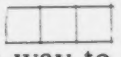
He left me curious about the simplicity of this task for school age children.

PERMUTATIONS
Judith K. Hall

MATERIAL: one die, notebook, pencil, calculator (optional), scratch paper

TIME: 2 weeks

MATH CONCEPTS: number order, place value, meaning of a digit, comparison of numbers, multiplying

PROCEDURE: I drew three boxes on the board, and had students do the same in their notebooks.  I rolled the die three times. Students placed the three numbers in a way to achieve the largest number... smallest number. I listed all the possibilities the children came up with. Students then generated combinations no one had made. After awhile, someone noticed you could only make six possible combinations -- if repeats of a number were not allowed. We tested this theory three times and agreed it was true.

I asked what would happen with four boxes. There was some guessing and I broke the class into groups of 4, had them draw 4 boxes in their notebook, and rolled the die. Once they had made their numbers, we wrote them on the board. They then listed all the combinations of that 4-digit number.

Lists were pooled and recorded in a table on the board. The table stayed there throughout the project so that information could be added or deleted. The presence of the table helped keep the project in the children's memory, and they worked on their findings at various times during the day. The table looked like this:

# of boxes	possible combinations
3	6
4	24

I then asked for predictions for 5 boxes, which they wrote in their notebook. Again, students drew 5 boxes, I shook the die, and they began to make the combinations and record them. It didn't take them long to see that there were going to be a lot of combinations. As a result, they began to delegate work within the group which worked out quite nicely. It took them two class periods to complete and agree on 120 combinations.

I asked if we could predict what would happen for 6, 7, or even 8 boxes based upon our previous knowledge. Groans were promptly heard, for they knew there would be a large number. Finally they began to look for patterns in the other solutions, and theories began to develop. The chart began to develop as follows:

# of boxes	possible combinations
3	6
4	24
5	120
6	?
7	?
8	?

The next task was to develop and prove a solution. I also asked for the solution to be put into words on paper.

Three class periods and evenings were spent working in their respective groups on solutions. The fourth class period, a Friday, was spent sharing and discussing different paths groups were taking. I asked them to continue working over the weekend. By Monday, this table had formed, and had been agreed upon by the entire class, even though they could not explain why ... yet!

On Monday, the class seemed to agree that the solution was to multiply the last combination by the next number of boxes, which would yield the number of possible combinations for that number of boxes.

# of boxes	possible combinations
3	6
4	24
5	120
6	720
7	5,040
8	40,320

One boy, though, had found another solution. He suggested that you multiply the number of boxes by the decreasing digits and that would give you the possible combinations. He didn't know why it worked, but it seemed to. We spent the remaining time in the period checking both solutions to see if they would work. The following calculations began to appear on the board:

# of boxes	pc	# of boxes	pc
3 X 2 X 1	6	3	6
4 X 3 X 2 X 1	24	4	24
5 X 4 X 3 X 2 X 1	120	5	120
6 X 5 X 4 X 3 X 2 X 1	720	6	720
		7	5,040

The following day, another child brought in a calculator manual she had been thumbing through. She had happened on a lesson on permutations, which stated the other child's rule! It's a factorial rule through which you can find the possible combinations of a number by using the x! key on a calculator. You enter the number of boxes and then press the x key and it gives you the possible combinations of that number!

We then used calculators to check our long, tediously worked calculations and found they were correct. Most were quite intrigued by this, and stated they had never even noticed that key on the calculator before. (I hadn't either!)

I then asked the students to write a journal type article in their notebooks describing the activity, their procedure, findings, and how they felt

in general about this kind of activity.

This was the first activity of this type I had attempted this year and was quite pleased with the interaction and outcomes.

The whole situation was easy for me, because it only required that the students have a notebook and pencil and I only needed one die, definitely not elaborate equipment.

The students benefited because it gave them an alternative to pencil and paper reviews at the beginning of the year. As this task progressed, I led a discussion on place value comma placement, and meaning of the digit in a certain column as we added boxes on.

The only part of the activity that was graded was the journal article. I had asked for certain information to be included -- state the problem, describe the steps, results, solutions, extensions and feelings towards the activity and told them to be creative. I also assigned two worksheets on place value as homework, and then gave a quiz after the activity on place value. I was quite pleased with the results of the articles, worksheets and quiz.

While the class worked, I took the opportunity to watch the interaction of the members of each group. I was quite pleased by their initiative to delegate work within the group, the quality of communication, the use of vocabulary heard in discussions I had led, and the seriousness of the work involvement.

Discipline problems never entered the sessions. When I asked the children to convey their attitudes toward this activity, the majority stated they liked this approach and would enjoy doing it again.

WRITING EXERCISES FOR MATH

Chuck Lugge

There are several ways to measure what our students are learning in math class. Tests, quizzes, and oral questions are only a few of the ways we assess their work. One method generally ignored in the math curriculum is writing exercises to gather information. While it takes precious time to read responses as opposed to checking answers to 2 + 4, essay answers allow for all the gray area between 100% correct and totally wrong. How much can you learn from a student's response of 2 + 4 = 8? Did he read the question correctly, mix up the sign or confuse the numbers? Has he always thought 2 + 4 = 8?

I realize that 2 + 4 = 8 is a simplistic example but I want to emphasize the utility of a writing exercise. You will have written record of the student's thought processes that you can review. In our example, our misguided student may have told us before that he confuses the signs, or that he had trouble concentrating on the simple problems. You can prescribe a corrective strategy directed toward that individual perception.

I'm suggesting the essay has its place in the math classroom along with the tests and worksheets. I have listed below some questions that could gather valuable information through written response:

1. Pretest: What do you know about X?
2. Post-test: What did you learn about X?
3. Values: What do you feel is the value of knowing about X?
5. Divergent thinking: How many different ways can you solve for X?
6. Write their own story problems: Can you think up a story problem that uses what we have learned about X?

These questions can assess what the student knows before the lesson, what he is learning during the instruction, whether he has internalized the concept or can organize it to use in a different situation, and finally, what he has gained from the instruction. The information gathered here may be even more reliable than tests with students who freeze up under pressure. You can use the essay to supplement or temper the grades you are extracting from testing.

I hope you see the potential and consider some alternatives to "Do page 123, every other problem."

AUTONOMY - ACTIVE PARTICIPATION
VS.
AUTHORITY STRUCTURE - APATHY

Sue Nash

While substitute teaching the last few months, I have been exposed to different classroom environments. Based upon some of my observations of these differences, I would like to suggest the following for thought. Mind you, I have done no scientific research, nor have I anything to base my thoughts on except my own observations.

Observation #1: A sixth grade class where everyone was held responsible for his own actions, including: going to the bathroom when needed, getting a drink when needed, talking with someone else in the class when needed, sharpening pencils when needed, working together on projects and/or assignments when asked by teacher and carrying one's own share of the load, etc. The student's only requirements were never to leave while the teacher was talking (unless it was an emergency), not to take advantage of freedoms by abusing them (following already established rules of conduct for bathroom, drinks, etc.), and to always keep in mind the welfare of others in the class. If a student abused his freedom, it was taken away until he felt he could conduct himself within the rules and could convince the teacher that he was sincere.

This class was a joy to teach. The students participated in all the discussions and lessons we covered. They took upon themselves the work of completing their assignments. They displayed respect for one another's feelings and property. There were no problems with angry kids picking on one another.

An interesting idea was a bulletin board covered with white paper. This was the graffiti corner, and students were allowed to write comments and names, draw pictures, etc., as long as they were not meant to hurt someone.

Observation #2: Five eighth grade classes dealing with the following subjects: Social Studies, English, Math. I was warned by the teacher's notes on her lesson plans that these students were difficult to deal with. The other teachers in the team warned me about the students' lack of cooperation. The lesson plans left by the teacher allowed little interaction of pupil - teacher.

They were basically, "Turn to page 96, do all problems, hand in at end of class," etc. (Perhaps, this was done for my benefit?) These students could, as the kids might say, "have cared less" about anything that had to do with school or learning.

As I reflected on my two day stay with these students, I considered what effect the rules of this school might have upon their attitudes. The day was scheduled so that these students had five minutes before school to get their books for the morning until 10:30. Then they had five minutes to get a drink, go to the bathroom, get their books for the classes that went until 12:00 noon. They were allowed twenty-five minutes for lunch in the school cafeteria, and then back to study hall or to chorus or band. At 1:00 classes began again and at 1:50 they had five minutes to get a drink, go to the restroom, get their books and be in class before the bell rang.

If a student did not accomplish all of these things within this time limit and had to raise his hand in front of all the students to ask to go to the bathroom he must stay after school for five minutes, after writing his name on the board for all other classes to see. I considered this demeaning to thirteen and fourteen year olds. By this age, one knows when his pencil needs sharpening, when he is thirsty, etc.

Had these students been respected as people when they were in the earlier grades and gradually allowed to become autonomous (responsible for their decisions and actions), I feel they would be able to live in the eighth grade without all these rules. These students have lost their desire to accomplish new tasks in the classroom: perhaps, because they aren't even given the right to try responsibly to go to the bathroom without penalty.

Surely, there were multiple factors which had contributed to the current state of mind of these eighth graders. I offer that if we would allow these students to make decisions about the above mentioned things, their self-esteem would rise and in turn they could change apathy into action in all areas of their lives. (For those students who, at first, can't handle this new freedom, take it away until the student feels he can assure the teacher that he is ready for a second try. We must trust until we are shown differently.)

The biggest barrier to all this seems to me to be getting permission from the principal of a building to use alternative methods with your students and not to be forced to accept rules blindly.

A SPECIAL PROBLEM

Loretta M. Kreke

I've been working on a problem offered in the summer course with David Lingard and Sylvia Johnson. The problem stated:

1) How many circles will fit around the center circle so that they just touch?
2) How many will fit on the next ring around?
3) And the next?
4) How many would fit if the center was larger than the ones to go around it?

I decided to use counters to work on my problem. I laid one in the middle and put a ring around it. I discovered six counters fit around the first ring. Twelve fit in the second ring, and eighteen in the next.

Ring	Number of counters
1	6
2	12
3	18
4	?

According to the chart the number is increasing by six. A possible formula:
Ring X 6 = Number of counters.

If this is true, there will be 24 counters in Ring 4. (4 X 6 = 24) I tried it, and there was. I feel the formula Ring X 6 = Number of Counters will work for all rings in this part of the problem.

The next part of the problem used a larger center. I used a quarter as my center and put the counters around it. Here is a chart of my results.

Ring	Number of counters
1	7
2	13
3	19
4	25

The number again increased by six, but the first ring was larger by 1. A possible formula is: (Ring X 6) + 1 = Number of counters

Now I wonder what would happen if the center was larger. This time I used a half dollar as my center. The following chart shows what happened this time.

Ring	Number of counters
1	8
2	14
3	20
4	26

The same thing happened, an increase by 6. And the center circle increases in size the number in the first ring increases, but after that the number increases by 6. A possible formula to include all the above charts would be (Ring X 6) + (Number in First Ring - 6) = Counters in any given ring.

This made me wonder what would happen if the center was smaller than my counters. I used a button with a diameter of one-half inch. (My counter diameter was three-fourth inch.) This chart shows my results.

Ring	Number of counters
1	5
2	11
3	17
4	23

The pattern of an increase of six continues. My formula from above will work in this case also.

I also tried this using all larger circles. This time I used half dollars. I got the same results as the chart on page one.

I feel that this increase of six for each ring will continue to be true. After each ring, there still is a circle, just larger. To go around each larger circle will continue to take six additional counters. Another interesting problem could be: What would happen using other geometric shapes.

DISTANCE COMPARISON

Shirley Gasparich

A fascinating Piagetian task involves marking off a specified distance, and walking the path from beginning to end, then reversing directions and returning to the start. Children may see the two distances as unequal as described in this article.

I worked with (23) 3rd graders (7 and 8 years old); (27) 5th graders (10 and 11 old); and several other children (8, 9, and 14 years old).

TASK:
1) Place 2 markers on the floor (any distance apart)
2) Walk from marker A or marker B using:
a) Giant Steps b) Baby Steps c) Same Size Steps

QUESTION: WHICH DISTANCE IS SHORTER? WHY?

I. THIRD GRADE CLASS (23 students)

* First, the students compared the distance using Giant Steps one way and Baby Steps the other.

A. 18 students thought giant steps = shorter distance because
a) Shorter time (faster)
b) Fewer number of steps taken

B. 2 students felt baby steps = shorter distance.

C. 3 students didn't seem to understand the task even after they had done it.

* Second, the students compared the distance using the same size steps each way, walking normally.

A. 9 students realized the distance was equal, but equated the = distance to

the = number of steps taken.

B. 4 students said the distances were not equal. Again these students attributed the inequality of the distances to an unequal number of steps taken each way. (4 - 5 steps vs. 6 - 7 steps)

C. 6 students didn't answer the question (time was short that day)

D. 4 students gave answers I was unable to interpret.

II. FIFTH GRADE CLASS (27 Students)

* First, the students compared the distance using Giant Steps vs. Baby Steps

A. 18 students felt Giant Steps = shorter distance because
a) Shorter time (faster)
b) Fewer number of steps taken

*** One of these 18 students changed her mind after discussing her findings with her group of 3 other students. She decided the distance was the same regardless of the size steps taken.

NOTE: EACH STUDENT PERFORMED THE TASK INDIVIDUALLY, ARRIVED AT, AND RECORDED HIS PERSONAL DECISION. THEN, HE DISCUSSED HIS FINDINGS WITH A GROUP OF 3 OTHER STUDENTS. STUDENTS WERE GIVEN THE OPTION OF CHANGING THEIR MINDS AFTER DISCUSSING THE ISSUE WITH OTHERS.

B. 7 students realized the equality of the distance regardless of whether baby or giant steps were taken. (They thought I had given them a "trick" question."

C. 2 students thought Baby Steps = shorter distance because:
a) "Because the giant step you put your feet farther apart."

b) "The shorter the step the farther you go."

*** My interpretation of (b) : Since with baby steps more steps are taken, it is, therefore, a longer distance. Or, it takes you longer, therefore, the distance is longer.

* Second, the students compared the distance using the same size steps each way, using a normal walking gait.

Of those who had time to complete his portion of the task, an overwhelming majority realized the equality of the distance using equal size steps from Marker A to Marker B or from Marker B to Marker A. However, many of them, although obtaining the correct answer, based their answer on the same number of steps taken rather than the equality of the distances.

*** Correct answers do not mean students understand if answers are based on faulty reasoning. The 3rd grade teacher could not believe that the students couldn't see the equality of using Giant Steps vs. Baby Steps over the same measuring line, especially after several had determined the distance was equal when using the same size step. So she gave them this example:

SALLY (STUDENT IN CLASS) IS LATE FOR SCHOOL SO SHE RUNS TO SCHOOL. SHE DOESN'T NEED TO HURRY HOME FOR ANY REASON, SO SHE JUST WALKS HOME AT HER NORMAL PACE. SHE TRAVELS THE VERY SAME WAY TO SCHOOL AND FROM SCHOOL TO HOME. WHICH IS THE SHORTER DISTANCE?

Everyone (23 students) said it was a shorter distance to school because she travelled the distance faster. (They equated distance to time or speed.)

I posed the same question to the 5th graders after they had completed the distance task and discussed their findings among both their group members and among the class as a whole.

13 students said the distance was equal no matter if she ran or walked.
12 students said it was a shorter distance if she ran because it would take

less time.
2 students had to leave class for lunch patrol.

Seven students had originally seen the equality of all the distances, arriving at their conclusions individually. Now, after group and class discussion six more students could see the equality of the distances. But 12 students were still confusing distance and speed or time even when I said it was 2 miles to school and 2 miles home. NOTE: In normal conversation distance is often spoken of in terms of time. A destination is 20 or 30 minutes away, rather than 10 - 12 miles away.

****

The 3rd grade teacher was beside herself in disbelief. She drew a line on the chalkboard and asked which way was shorter from A or B or from B to A? Most of the students gave the correct answer, but could not see the relationship of this information to Sally's running and walking to school!

****

In the 5th grade, one of the original 7 students, who had realized the equality of the distances, drew a similar line on the chalkboard, posing the question: IF YOU RUN (use giant steps) FROM A TO B AND USE BABY STEPS FROM B TO A WHICH DISTANCE IS SHORTER? The students who were confusing distance with time required to travel the distance still could not see the equality.

****

III. I also asked (6) randomly chosen 9 year olds (4th graders) to do this same task. None of them saw the equality of the distances, except when using the same size steps from A to B & from B to A. They all reasoned a shorter distance was travelled using giant steps because it took less time.

****

IV. I asked my 8 year old niece to do this same task. She looked at me strangely and said, without doing the task, "It's the same distance, no matter how you walk!"

****

V. My 14 year old son gave me the same response. I then asked him to compare the distance going 1 way in a rectangle, using small steps, vs. going the opposite way, using longer steps. He again said it didn't matter which way you went or the type of steps taken; both distances were equal.

****

OBSERVATIONS:

1) To both 8 and 14 year old, the equality of the distances was based on logical necessity. To them, the answer was obvious. Thus level of development, maturity, or perhaps experience and exposure appear more relevant here than age.

2) No wonder word problems pose such difficulty for students. Many of the students I worked with failed to see that distance and size of steps taken are not related. Certainly vocabulary is a problem, but I think seeing relationships among and between classes of things is a more difficult problem.

3) This task also seems to point out that many children even up to and including 5th graders (10 and 11 year olds) seem to have a class inclusion problem. They have difficulty seeing the relationship of parts (Giant Steps vs. Baby Steps) to whole (Distance comparison). Once the students focused or centered on the size & number of steps needed to transverse the space from A to B, they seemed to forget or were unable to realize the question was dealing with distance. (Which is the shorter distance?) Yet, when they gave the rationale for their answers, they used the word shorter distance, so they were equating: a) Time to

distance b) Number of steps taken to distance

4) Certainly, this task also seems to involve a problem with thinking simultaneously of whole and parts even though the parts (size of steps) are not really related to distance (whole).

5) The fact that some of the children could see the equality of going from A to B or from B to A when a line was drawn on the chalkboard seems to be an example of horizontal de'calage in which the child may display different levels of achievement regarding similar areas of thought.

6) This task emphasized both to the teachers I worked with and me the importance of not taking children's knowledge for granted (even correct answers). What seems so apparent to an adult or older student may not be apparent at all to a child who is at a different stage of development. Even when he discovered the equality of the distances in one instance, he was unable to generalize and transfer the constructed information to other circumstances.

7) Many students could see the equality of the distance A to B or B to A when a line was drawn on the chalkboard, emphasizing to me that the children could see the relationship as they are accustomed to seeing it (2 - Dimensionally), but not when it was presented as a real life problem (3 - Dimensionally).

****SHOULDN'T classroom math be dealing with real life issues rather than with paper & pencil tasks?

8) Finally, vocabulary is very important! The student's definition & inner construct of distance may be very different from an adult's view.

Certainly, attending to and coordinating several aspects of a problem simultaneously, eliminating necessary & unnecessary information, & generalizing from 2-Dimensional to 3-Dimensional problems are very difficult tasks for children --- obviously even through 5th grade.

BRAINSTORMING IDEAS OR HOW CAN I USE THIS "JUNK?"

Barbara Goldenhersh

After experiencing the activities David Lingard and Sylvia Johnson shared in their summer course, they gave us the chance to practice expanding our horizons by dealing with the type of materials we might have on hand. What follows are the thoughts we came up with in just a few minutes of freeflowing brainstorming . What other ideas can you devise?

COLORED CHIPS

1. How many sequences can you make with 2 colors? 3? 4?
2. How many different colors can you make if the chips are transparent?
3. Can you make a color wheel?
4. How many chips will fit around another?
5. How many counters do you have?
6. Can you make tesselations? How many?
7. Overlap any 2, 3 ..., Look at the shapes of intersections?
8. How many original colors are there? Name them.
9. Can you use the chips to figure the area.
10. How many figures can you make with "8" counters?
11. Given 4 different colors, what possible ways can they be arranged?
12. Develop a board game.
13. Can you use them for money? Poker chips? What else?
14. Can you develop a maze?
15. Using a specified number of counters, make a square of the same color with one blue one and one empty space. Picking a destination for the blue one, how many moves will it take to get there?
16. How many Geometric shapes can you make? How can you change shapes in x number of moves?
17. How many letters can you make?
18. How many circular figures?
19. If 10 are dropped, how many would touch?
20. How many counters in 6 feet?
21. How many punctuation marks are there?
22. How many math figures?

23. How many can you pile up without them falling over?
24. What is the circumference?
25. What is the weight?
26. Relate designs to number patterns.
27. How many triangles can be made within a triangle?
28. How many chips will fill certain spaces?
29. How many counters are there in an 8 counter equilateral triangle?
30. How many counters fit around a 6 inch circle?
31. Use to teach place value.
32. How far can you roll one?
33. How far can you tiddly-wink one?
34. How many are needed to stand one up?
35. Can you do spinning tricks?
36. Teach primary colors.
37. Make stair steps.
38. With base of 40 counters - How many chips are needed to make a triangle? Are there different solutions? What others can you find?
39. Chart a graph comparing quantity (#) by color.
40. If stacked, how high? How many are in your stack? How high would 85 measure?
41. How many different shapes can you make using counters ? How many counters in each shape?
42. If you have 5 counters as a base, how many are needed to make an equilateral triangle? 6, 7, 8, 9, 10?
43. Can you make a pattern in which each counter touches 2 other counters? 3, 4, 5 When does it stop working? How many were touching at that point?
44. If your triangle has 6 counters, how many moves must you make to invert your triangle?
45. Is there a number that will not make a triangle?
46. Describe the hidden disc without telling its color.
47. Can I play tiddly winks?
48. Can I play bingo?
49. Can you make_____?
50. Make a 3 x 8 enclosed shape. How many spaces inside? Can you use the same chips to form a shape with fewer spaces?
51. Given a pattern of different colored chips, can you predict what will come next?

52. How many counters can touch one counter at a time?
53. What's the greatest area you can enclose with all your counters?
54. Surround one counter. How many counters were needed? Put a second row. How many counters? Repeat. Is there a pattern?
55. Make a triangle. Add the fewest number of chips to make a larger triangle. How many chips do you need to make a 10 row triangle?
56. Given a certain number of chips, can you form a triangle that will use all of the chips? If not, what is the largest number you can use?

PAPER STRIPS

1. How many different shapes are there?
2. How many circles of various radius?
3. How many 3 dimensional cubes can be made?
4. What math symbols can be made?
5. How many are slanted when piled?
6. How many angles can be made?
7. What is the shortest distance between points?
8. What is the strongest figure you can build?
9. What is the largest circumference?
10. What is the largest volume?
11. How many different figures are there with same perimeter?
12. How many shapes are there? What are the shapes? What's the most sided shape you could make?
13. How is force related to length?
14. How many letters of the alphabet can be made. What are they?
15. How long are the strips? What's the longest? How many lengths? What's the shortest?
16. What is the maximum length of all 5 together?
17. Given the weight of 2 - calculate the weight of 5.
18. How wide are they?
19. How many are there?
20. How many intersections? How many intersections use 2...3...4...5...6? What's the most? What's the least?
21. Can you develop a spiral? What would be the longest spiral?
22. What 3 -D figures could you make?
23. What's the area? What shapes give the greatest area? The least?

24. What kind of angles could you make?
25. Could you use the strips to make a graph? How?
26. What familiar figures can you make?
27. What's the longest possible line?
28. How many crossings are there?
29. What structure(s) can you make that won't fall apart?
30. How many ways can you group them?
31. Can I bend the strip to make something?
32. What shapes are made by the angles of intersecting lines?
33. Can you make 4 obtuse angles? Using 2,3,4, 5 strips?
34. Can you make 4 acute angles? Using 2,3,4,5 strips?
35. How many different shapes can you make? Using 2,3,4,5 strips?
36. How many angles can you make? Using 2,3,4,5 strips?
37. Pick up all the strips and drop randomly. Look for angles formed and/or intersecting lines?
38. What is the largest perimeter that can be made from all the strips? Shortest?
39. What is the largest a rea you can enclose with them?
40. What shape does the intersection of 2 strips make?
41. Can you make a square?
42. Can you make a math's equation?
43. Can you use all of them to make a 90 degree triangle? If not, how many would you need?
44. What numbers can you make with them? (Including Roman Numerals)
45. How my different symmetrical patterns can you make?
46. Can you make a cube from these 5 strips? Do you need more strips? How many more to make a cube?
47. How many vertices can you make?
48. What is the longest shape you can make?
49. What is the shortest shape you can make?
50. Can you make a free standing structure?
51. If we could tape them all together what would the largest circumference be with 5...4...3...2...1?
52. What shape would have the largest area? The smallest area?
53. What is the area of a pentagon?
54. If you had 70 of these, how large a circle could you make? What is its circumference? Area? How large a square? What is its area? How large a rectangle? What is its area?

55. Using 5 strips, can you make a 120 degree angle?
56. How many letters of the alphabet can you make out of 5 strips?
57. What can you make with 1 strip? 2? 3? 4?
58. How many wraps around a pencil?
59. If the paper strips represented lines, how could you divide a piece of paper? (Using 1...2...3...)

DOTTY PAPER (sample paper is included in this issue)

1. What is the distance between dots?
2. How many ways from one point to another?
   a. shortest? longest?
   b. how many right angles are there?
3. If you have 3 X 3 dots, how many triangles can be formed?
4. Using an enclosed 4 X 5 square space show 2/5, 1/4, 1/5.
5. How many dots are on the page?
6. How many circles can we draw within a 6 X 6 area? How many will not overlap?
7. Can you make 6 different shapes within a 6 X 6 area jigsaw style without overlapping?
8. Develop a maze.
9. How many squares are there?
10. Which triangle is largest? Equilateral? Isosceles? Is each of these possible?
11. Given a 3 X 3 array, How many ways are there to divide in half?
12. On a 3 X 6 array, can you fit in 6 different shapes with the same area?
13. How many different equilateral figures can you make?
14. What fraction of the paper is covered in dots?
15. Given 9 dots, how many triangles can you make?
16. How many different figures can you make?
17. What is the area of white to black?
18. Using rubber bands - how many dots are encircled?
19. Figure the areas of different shapes.
20. Figure the perimeter of different shapes.
21. How many cubes are on the paper?
22. Fold to make the most various shapes.
23. How many triangles can be made?

24. Connecting 20 dots, which area is most enclosed?
25 . Build the strongest structure.
26. Do factor studies.
27. Draw specific animal figures with dots.
28. Play games by connecting lines.
29. Draw 3 dimensional figures.
30. Probability studies (Drop Pen).
31. How many different shapes can you make?
32. How many lines of 5 dots?
33. What's the largest square on the page?
34. How many squares are on the paper?
35. Find the area of a scalene triangle not in line with the dots.
36. How many dots are in a line? diagonally?
37. Can you draw a straight line from 2 corners?
38. If you use an area o f 10 X 10, label top left A, bottom right B, how many moves to get from A t o B? Shortest? Longest? What are all the possible paths?
39. Can stair steps be made from top to bottom of the page?
40. How many different sized stai r steps are there?
41. What size boxes can you make?
42. If I draw a circle with its center on a dot and a pencil on another dot, how many dots will the circumference go through? Change the radius, will you always cross the same number of dots?
43. Draw a figure connecting 6 dots on your paper to form a closed figure. Can you tesselate?

THE STUDENTS EXAMINE SURFACE AREA

Barbara Madden

Last Monday morning when the kids returned from weekends of ball practice, bike riding, camping, or tramping in the fields or woods, they were sunburned and tired, but there was an air of excitement - only 15 more school days. As the time approached for first hour class, my eighth grade students drifted into the classroom. The students found their places. The bell rang. The excitement was gone.

With the exception of three or four students, first hour class was quiet and unresponsive. Second hour was equally quiet. I was sure I was "brilliant" until I looked at the kids. There had to be a change in third hour. It was time for me to keep quiet and allow the students to be "brilliant."

That day we were to study surface area of solids. The students had had some experience with surface area in previous years. This was to be an extension of what they already knew. When class began I abandoned my traditional lesson plan. In a brief discussion we reached agreement on the meaning of surface area. I allowed the class to divide themselves into groups of two or three and I placed my large box of geometric solids in the center of the room. Groups were to choose a solid, measure it, find surface area, record their procedure and their answer. At the end of the period each group was to report their findings on one shape to the rest of the class .

The excitement returned to the students. They formed their groups. One girl who is normally aloof in the class was invited to join a group. She smiled and became involved. I walked around the room. I listened, I suggested, I encouraged, I enjoyed! Students argued their points and defended their strategies. On several occasions they had to clarify their thinking on the difference between volume and surface area. One group puzzled over the area of a hexagon. Finally, they decided to divide it into two congruent trapezoids. They knew how to find the area of a trapezoid. Another group, finding the surface area of a hexagonal pyramid, divided the base into six congruent triangles. The students were pleased with themselves. Each group had found

the surface area of three or four shapes.

At the end of the period a student from each group explained a method for finding the surface area of one solid. Communicating their procedures was very difficult for most of the students. I'm not sure how many of the students understood what was said, but they were attentive and respectful. The difficulty in communication is all the more reason to repeat this process with the students.

Jeff chose to work alone. He asked for a "hard" shape. I suggested he try to find the surface area of a cone. The students had made cones and cylinders from construction paper a few days before. I suggested he recall that experience.

He worked for the entire period. He reasoned that he'd have to use a proportion, but he wasn't able to write the proportion by the time the bell rang. From time to time during the day he popped in to report his progress. The next morning he met me as I entered the building. He showed me his proportion: circumference of base of cone; circumference of original circle = lateral area of cone; area of original circle. He had solved the problem. He was excited. I was excited.

When I told my principal of this really neat experience; he expressed concern that Jeff was "always working during lunch."

The following Monday Jeff asked how to find the area under a curve. He was trying to find the area of the ellipsoid he had seen in my box of solids. We began talking about limits and the Riemann sum. Jeff is challenged and so am I!

IS THIS WHAT YOU CALL EDUCATION?!?

Jane Stump

It's August 21, and a teacher works to get everything just right for her incoming class of second graders. The calendar boasts huge, red, numbered apples, Charlie Brown and Snoopy hand-spring across a bulletin board, and prominently displayed is this poster:

C A U T I O N
human beings here
handle with care

"Risky," thought the teacher, "but I've got to try it. The kids deserve better material than what's here in these books." Repetitiously she counted out a text and a workbook on each of the 29 desks for three subjects - reading, English, and math; then she topped each stack with science, spelling, and social studies texts. "The district expects seven-year-olds to ..." She stopped. "It's riskier than I thought ... and there's only two more days to work it out." Methodically, she went back around the room, collecting the math and science books, knowing the other books would come out of the desks only when necessary. Then she put the desks in groups of six, freeing what seemed to be a "huge" amount of floor space adjacent to a blackboard. "For my rug," she mused.

She settled in with her planbook, erasing frantically.

Two days later, 29 faces of assorted shapes and colors found their room. Some walked around the classroom, looking at the wildlife and plant posters. Some looked in astonishment at the six-foot tall rubber plant, and others gathered around the bubbling fishtank. A few sat down at desks and chatted excitedly. As the teacher moved into the room from her post by the door, she overheard a pair of bouncy pigtails saying, "This looks like a grown-up kindergarten, or something." "Yeah," replied a nearby set of wide, brown eyes, who thought for a moment before quietly questioning, "but I wonder what we'll do."

"Now it's certain!" decided the teacher. "There's no turning back."

Gathering the children together on the bright orange rug, she set about her task.

Despite some rough spots, it all began to fit into place - math was done with bottle tops, beans, and colored macaroni. ("How I wish the school had Cuisinaire Rods, or at least Flexicubes," she thought.) Science was all around them - and the kids were so involved in discovering how the plants and animals prepared for autumn. They readily "scienced" the woods along the playground's edge. The English came into place as records of their observations, questions and discoveries were made. The reading and the social studies made so much sense, and took their place, too, when the kids wanted to find out "more".

Three months later on a cold Monday morning, it happened. Her very own child awoke with an upset stomach and a fever of 103°. What else could she do?

A sub was called. One week and several gray hairs later, she went away.

Behind her, the sub left five neat rows of separate desks, ten pages of math and English per child, graded. The rug was rolled up and leaning in a corner, and in the fishtank, a goldfish floated lifelessly. She left this note:

> *What are you doing in here? When I came in on Monday, the kids were all over the room --- poking at the plants, sprawling on the floor with all sorts of books, and piling kidney beans under margarine tubs.*
>
> *Do you call this education?*

And while the teacher regrouped the desks and unrolled the rug, she paused to look out of the window. There, on the playground, were all of "her bouncy pigtails and wide, brown eyes. With more determination than ever she turned towards her desk, and filing the sub's note for posterity, chuckled:

"Yep. It's worth every bit of the risk!"

POINTS AND CIRCLES

Maureen Schreiber

Introducing a unit of geometry to a group of eighth graders was my opportunity to pose to the students several problems which were presented during the Summer Workshop at SIUE's Teachers' Center Project.

As an introduction I held my index finger in the air and told the students I was leaving a point suspended in the air. We agreed that I could put points at any spot. When I put a point on the chalkboard, I started to erase as much chalk as possible to be sure the students understood that a point had no dimension.

Next we discussed the terminology of circumference of a circle. When I asked the class how many points could I draw on the circumference of a circle, I received the expected answer of 360. Again we discussed that the answer of 360 came from 360 degrees in a circle. Eventually the conclusion came that we could draw an infinite amount of points on a circle.

After asking the students to draw a point on their paper, I asked the class to draw a circle with the circumference of the circle going through the point. A second circle should be drawn with the circumference also through the same point. A third circle was drawn with the same requirement.

The students had a variety of answers and seemed content that although some had different pictures, all the answers were following the directions given. I was surprised that they weren't worried about the "one right answer."

The students then started to direct me to draw on the chalkboard their combinations of circles. We decided that several pictures were just reversals or rotations of others.

Two points were then drawn on a piece of paper. The students were requested to draw a circle with the circumference of the circle passing through both points. I had expected to see many students placing the points on the diagonal of the circle. This placement did occur, but not to the amount I expected. When I asked the students to draw a second circle, some students could not achieve this task. One bright girl explained, "It can't be done!" After they saw a second circle drawn, some students could draw the third circle by reflection, but could not see how to draw a fourth circle and so forth. Eventually the students realized that again with two points an infinite amount of circles could be drawn.

Three points were then drawn on the piece of paper. I expected many students to be unable to draw this circle. Surprisingly many students placed their points at the vertices of an equilateral triangle and could find a circle. When I placed three points on the chalkboard, not at any regular placement, still some

students could find a circle through these points, however they did not seem to have any particular method of finding this circle and some drawings did not look like an accurate circle. An explanation was given on finding chords and bisecting the chords to find the center of the circle.

Throughout this lesson, the class remained as a large group and did not seem to be motivated to work in small groups. I tried to get some students to try to find if they could make more than one circle through any three points. They were not motivated, so I agreed that only one circle could be drawn through three points and asked if they would draw four points. Many students were loosing interest now in the problem and I could see it was time to change problems. We all came to the conclusion that only one circle could be drawn through four or more points and those points would have to be in particular positions for this circle to be drawn.

The next problem to solve involved circles again. This time we worked with combining circles. I had passed out paper circles to the class of varying sizes. I asked each student to draw three circles on their paper in any position. Again I had the students direct me to draw on the chalkboard their combinations. After having several drawings, we again discussed duplications to eliminate. How many different combinations of three circles could they make?

The class acted as if this task was insurmountable. They were allowed to reduce the qualifications. "Lets use only one circle!" We agreed to this and found of course only one answer. Next the students wanted to use two circles and accept this as their homework assignment. I encouraged them to try to accomplish more by explaining that a four year old child has accomplished the task of looking at two circles. the class still did not want to look at the possible combinations of three circles of three possible sizes. They could not seem to come to any more qualifications other than the number of circles.

Finally, because of time, I had to end the discussion. I made another qualification and asked the class to work on looking at the combinations of three circles of the same size. Since I would not be present in the class the following day, I asked the students how many of them would actually do the assignment if it had not been assigned as homework. Not one student volunteered to work it out. I wonder if in reality someone would have ventured to solve the problem on their own.

PLACE VALUE PRESENTED CONCRETELY

Linda Pauk

THE PIAGET PRIMER by Ed Labinowicz suggests that many students are presented with base ten blocks for both place value and regrouping but most students do not have a real understanding of what these blocks represent. The author suggests the use of beans and muffin papers as a concrete introductory activity to give real meaning to the base ten blocks. I would like to share some activities I have used to present base ten concepts concretely.

Students are first given groups of beans to count. Muffin papers can then be introduced as a way to help the counting. I suggest writing "ten" in the bottom of each muffin paper to remind the student how many beans to place in the paper and to begin the vocabulary associated with base ten concepts. The students now count their beans and take the extra step of placing ten beans in each muffin paper. Counting by ten will quickly replace the individual counting of the beginning activity. After successful practice of counting using the muffin papers, introduce a jar with a lid and label this jar as "hundred". The students now can discover that ten of the muffin papers filled with beans will equal 100 beans which can now be place in the container with the value of 100. Although time consuming, practice time using several hundred beans will give the students the concrete practice they need before introducing the standard base ten blocks. It is also important to present written numbers and have the students form these combinations with the beans, papers and jars. As these activities progress, much emphasis should be placed on calling the groups by the names of ones, tens and hundreds. When the students are very comfortable with using beans for place value, the standard base ten blocks can be introduced as a more compact representation of what the beans have been showing. The students show a much greater understanding of what is represented by the base ten blocks.

Regrouping for both addition and subtraction will be understood much sooner if initial activities include use of the beans, papers and jars. For addition, the students are asked to place a given number of beans in an open

box. They are then asked to add another number of beans to the box. Since they have already had a great deal of experience with the ones, tens and hundreds, the students will form these numbers using beans, papers and jars. As the students analyze the total number in the box they notice that there are now enough single beans to form a new muffin paper and therefore concretely act out regrouping. Again, the teacher can be introducing regrouping vocabulary as the students manipulate the beans. Only after several sessions of bean practice, is the introductory work with written problems presented.

Subtraction with regrouping can also become more understandable by using the beans and a box. The students are asked to place a given number of beans in the box and are then requested to take away another number of beans. Since the students will be using muffin papers and single beans, they will find that they cannot remove the requested number of beans without breaking up a muffin paper of beans and adding that ten to the single beans in the box. Again, gradually introduce vocabulary and transfer the activity from beans to base ten blocks. The beans are much easier to manipulate at the beginning because a muffin paper can simply be poured out rather than having to go somewhere else and trade for different blocks as you have to with base ten blocks.

Although these activities may seem time consuming, the understanding gained by this time will be invaluable as you progress into more difficult areas. Many of us cut back on basic activities for the lack of time and find ourselves spending much longer on tasks because of lack of understanding. Always try to approach the task in the most concrete way possible and success will follow.

SUBORDINATION OF TEACHING TO LEARNING
Reduce Math Anxiety ... Promote Thinking!
Shirley Gasparich

As an adult education (GED) instructor, I have seen many students who for one reason or another have failed to thrive in our present educational system. Their confidence in their abilities to learn "school work" -- math in particular -- is extremely shakey because of their past school failures and/or because of the long number of.years they have been out of school. However, these adults share traits common to all students, primary through college. They need to feel a sense of success and self-confidence in their abilities to achieve, or they will not persist in their efforts to learn.

Noted mathematician and educator, Hassler Whitney states that remedial work, with its tutoring, practicing, and more practicing, seldom achieves real progress, but instead elevates the student's feeling of insecurity and failure. To be sure, tutoring is beneficial initially, but the student fears that his poor memory will reappear, and in time, he will be "dumb" once again. To avoid this, the child/adult tries harder. He tries to remember: "What should I say here?" "What is the answer?" Instead of thinking about what to do, he feels safer at guessing what to say. Whitney points out that as educators, our goals need to be: to shift the student's objective from getting an answer to gaining confidence in looking for an answer!

Recently, my students were experiencing a full blown case of "dumbness" characterized by blank expressions and frustration. We were working the following percent problem:

Glenda bought maple syrup for $1.92 a pint and sold it for $ .96 a pint more. By what percent did she mark up the price of maple syrup?

The formula R X B = P had been presented and explained to my complete satisfaction, yet, only one student was able to work the problem by plugging the necessary numbers into the formula. Worse yet, the others didn't even know if their answers were reasonable.

Finally, I asked what the students could change $1.92 to? Several students suggested it was almost $2.00. Then, someone said, "$ .96 is almost

$1.00." I then asked "What is the percent of mark up using these numbers?" Instantly, without pencil, paper, or formula, even my slowest student shouted -- "There is a 50% mark up!"

The classroom felt like Times Square on New Years Eve! There was an exciting electrical energy present that one would not have imagined possible thirty seconds earlier. The students, who had been frustrated and ready to quit moments earlier, felt they had discovered a new found tool -- their own ability to **THINK!!**

I, too, relearned a valuable lesson -- the importance of subordinating teaching to learning as Caleb Gatteneo so succinctly puts it.[1] I was also reminded of Hassler Whitney's work with Elizabeth, a remedial, middle grade student. Instead of showing her what to do, he challenged her to think for herself.

As a classroom teacher, I was reminded that formulas and pat techniques do not work if the students have no insight into the problem. The students must interact, make a connection, construct personal meanings to the words and numbers, or otherwise they remain passive bystanders. Too, we educators must be ever mindful -- "math does not have to be difficult if the right learning methods (as opposed to teaching methods) are used."[2]

1Caleb Gatteneo, ***WHAT WE OWE CHILDREN****, Outerbridge and Dienstfrey, New York, NY.*

*2****MATHEMATICS IN PRIMARY SCHOOLS****, Schools Council Curriculum Bulletin No. 1, fourth edition, page 9.*

THE SUM OF WHOLE NUMBERS FROM ONE TO ONE HUNDRED

Barbara Madden

I came upon this problem in a recent mathematics journal. The teacher had asked the class to find the sum of the numbers from one to 100. Gauss had the answer by the time the teacher had finished asking the question. My students were not quite so fast

The students worked in groups. Their strategies were diverse. One group began writing the numbers from 1 to 100 in long columns and planned to add the columns. Another group separated the numbers into even and odd. Both methods were quickly abandoned.

A third group organized the numbers in 10 columns of 10 numbers each.

1	2	3	4	...	10
11	12	13	14	...	20
21	22	23	24	...	30
31	32	33	34	...	40
41	42	43	44	...	50
51	52	53	54	...	60
61	62	63	64	...	70
71	72	73	74	...	80
81	82	83	84	...	90
91	92	93	94	..	100
460	470	480	490	...	550

The strategy was to add each column, and then add these sums. The students soon recognized a pattern. Each column differed by ten. They were

able to look back to see why this was so. Most students found the correct total of 5,050.

The next day we again discussed the problem. I told them the story of Gauss. They wondered how he had solved the problem so quickly.

I showed them this technique with the sum of numbers from one to ten. Successively add:

1 + 10 = 11
2 + 9 = 11
3 + 8 = 11
4 + 7 = 11
5 + 6 = 11

and five sums of eleven is 55.

The students extended this process to 50 and finally to 100.

One student asked if it would work for 1,000. I said, "Try it!" He did. Five hundred sums of 1,001 is 500,500.

GROUP PROBLEM SOLVING

The following articles are attempts by participants in the summer course to develop their own group problem solving activities for their classrooms. We hope you will try these out as well as develop your own. Barbara Goldenhersh -- editor

GROUP PROBLEM SOLVING
Jean Anderson

CLUE SHEET #1

Sam, Tom, and Jerry each had 3 errands to run after school. They all left school when the last bell rang.

How many blocks did each travel?
How much money did each spend?
Who spent the most time? Money?
Who walked the least? Sat most?

1. Sam went to the Library after his dentist appointment.
2. Bread costs $1.00.
3. Tom had to go to buy music at the Music Store before his lesson.
4. Soccer practice is at the High School.
5. Sam bought a $.25 toothbrush at the dentist.
6. Jerry went to soccer practice after school.
7. Tom paid $1.00 for his lesson.

CLUE SHEET #2

Sam, Tom, and Jerry each had 3 errands to run after school. They all left school when the last bell rang.

How many blocks did each travel?
How much money did each spend?

Who spent the most time? Money?
Who walked least? Sat most?

1. Jerry's last stop was the Bicycle Shop to pick up his bike.
2. Sam had to take a book back to the Library.
3. Tom stopped at the Ice Cream Shop for a cone after his music lesson.
4. Socks cost $.75.
5. The bus costs $.50.
6. Before going to soccer, Jerry bought a pair of socks at the Sport Shop.
7. Cones cost $.25.

CLUE SHEET #3

Sam, Tom, and Jerry each had 3 errands to run after school. They all left school when the last bell rang.

How many blocks did each travel?
How much money did each spend?
Who spent the most time? Money?
Who walked least? Sat most?

1. Sam's last stop was the Grocery Store for a loaf of bread.
2. Sam rode the bus to the dentist office.
3. Music cost $1.25.
4. There was a $.10 Library fine.
5. Jerry's bike repair cost $2.00.
6. Tom had a drum lesson at the High School.
7. Sam had a dentist appointment after school. He went there first.
8. Jerry had to pay a soccer fee of $.50.

Sport Shop
Sam's House
School
bus
Library
Grocery Store
High School
music Store
Bicycle Shop
ice cream shop
Dentist
Jerry's House
Tom's House

GROUP PROBLEM SOLVING
Bob Moran

THE PARK PROBLEM Clue Sheet 1

The problem facing the Jones family is which route they will take home after school is dismissed for the day. Each route has numerous obstacles and various games which mark the way.

There is an ice cream parlor which sells ice cream cones for ten cents each. Amy Jones going this way will buy two cones. It takes three minutes to purchase the cones.

Pin-ball machines are one of your favorite games. You play three games at five cents a game. You also win two free games. Each game takes two minutes and 15 seconds to play.

The wall on the other side of the stream is six feet high. To climb the wall you must be able to reach the top and pull yourself over. It takes one-fifth of an hour to climb the wall.

John Jones is 4'8" tall and weights 115 lbs.

Question: 1) Which is the fastest route home? 2) Which of the Jones' family are you?

THE PARK PROBLEM Clue Sheet 2

The problem facing the Jones family is which route they will take home after school is dismissed for the day. Each route has numerous obstacles and various games which mark the way.

The swings are broken and must be fixed before they can be used. It takes six minutes to fix all three swings.

At the bridge you see your uncle Phil who is fishing. You help him clean three fish. Each fish takes forty seconds.

The trees around the stream have a rope attached to a limb so you can swing across. The maximum weight the rope will hold is 120 lbs. It takes three minutes to get the rope and swing across.

If you run home you can travel one block in two minutes. However, you fall down and cut your leg. It takes one-sixth of an hour for you to wash and put something on it.

Tom Jones is 4'0" and weighs 125 lbs.

Question: How much money will you spend?

THE PARK PROBLEM Clue Sheet 3

The problem facing the Jones family is which route they will take home after school is dismissed for the day. Each route has numerous obstacles and various games which mark the way.

The tunnel has a long line of your playmates waiting to use it. It takes ten minutes to use the tunnel.

The candy store is still open. You have enough money to buy three candy bars at five cents each. It takes five minutes to make your selection.

After crossing the bridge you drop fifty cents in the sewer. It takes you twice as long to get the money from the sewer as it did to buy the ice cream cones.

If you ride your bike home you can go one block in one minute. However, you have a flat tire and it takes you eighteen minutes to repair it.

Judy Jones is 3'6" tall and weighs 95 lbs.

Question: What is the route you will take home?

THE PARK PROBLEM Clue Sheet 4

The problem facing the Jones family is which route they will take home after school is dismissed for the day. Each route has numerous obstacles and various games which mark the way.

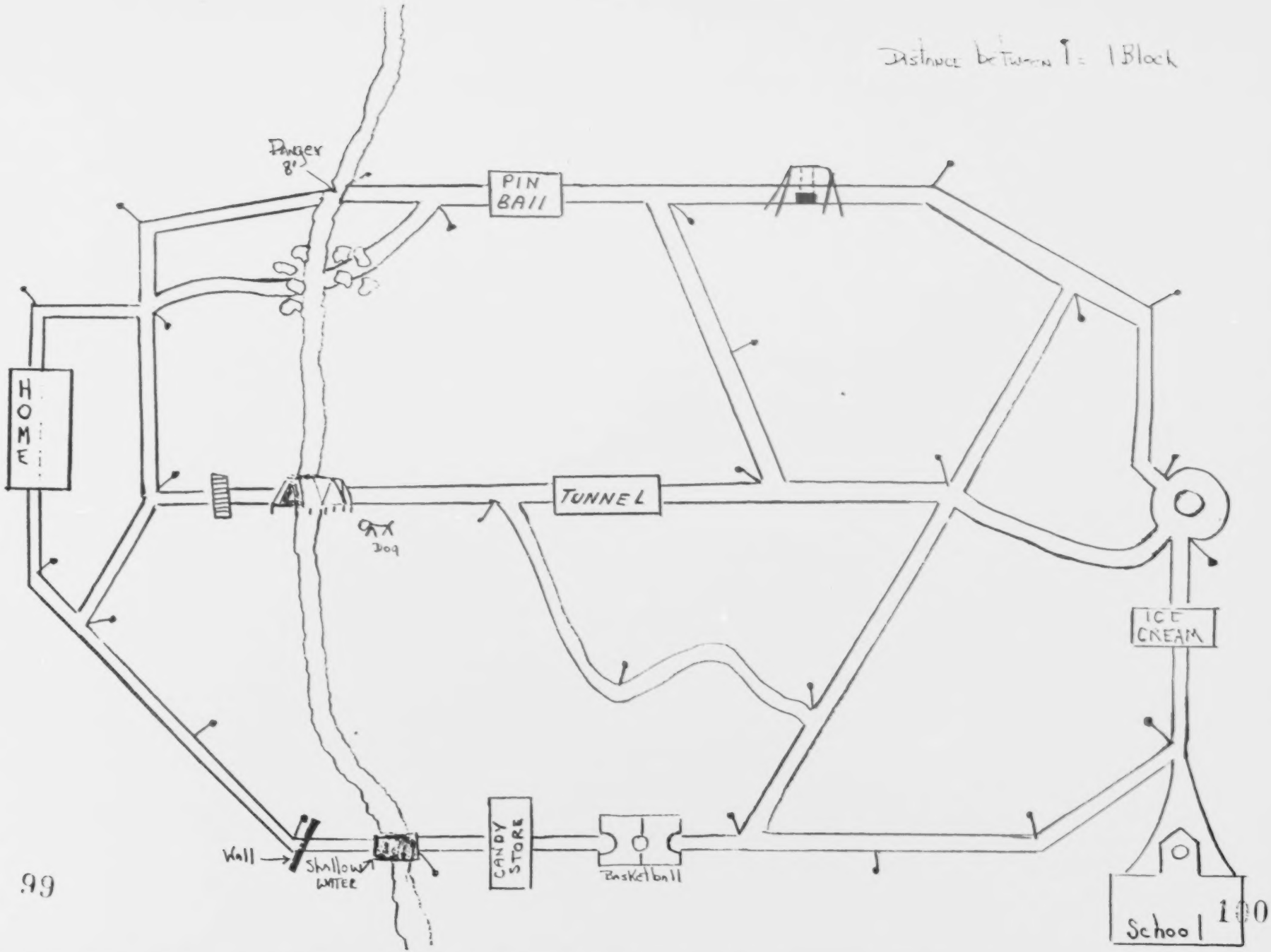
There is a basketball game taking place in the park. Any of the Jones family going this way will stop and play basketball for fifteen minutes.

As you near the bridge you encounter a dog who appears to be vicious. You must wait twenty-five minutes until the owner comes to get him.

If you walk home it takes you one-half of an hour to walk one block.

School is released at 3:00 p.m.

Question: What time will you arrive home?


Distance between T = 1 Block
Danger 8'
PIN BALL
HOME
TUNNEL
Dog
ICE CREAM
Wall
Shallow WATER
CANDY STORE
Basketball
School

GROUP PROBLEM SOLVING

Tracy Brubeck

CLUE SHEET #1

Your group is in charge of planning the refreshments for a class party for the graduating 6th grade class. You have 30 dollars to spend. The party will be on Saturday, June 7. On June 1st the merchants of Belleville are all participating in a huge sidewalk sale. Your group has decided to do all its shopping on this day because of the special sales.

K-Mart has soda on sale for $1.08 per 6-pack.

Schnucks and Kroger are both running cases (24 cans) for $4.80

National and Wal Mart have 12 packs for $2.40

Where will you buy your sodas and how much will it cost?

CLUE SHEET #2

Your group is in charge of planning the refreshments for a class party for the graduating 6th grade class you have 30 dollars to spend. The party will be on Saturday, June 7. On June 1st the merchants of Belleville are all participating in a huge sidewalk sale. Your group had decided to do all its shopping on this day because of all the special sales.

There will be 45 students at the party and 10 teachers. You must provide 2 cans of soda for each person. You've decided you want at least 3 different types of chips for variety. You've figured you must get one 28 oz. bag of chips for every 2 people. How many sodas and bags of chips did you buy?

CLUE SHEET #3

Your group is in charge of planning the refreshments for a class party for the graduating 6th grade class. You have 30 dollars to spend. The party will be on Saturday, June 7. On June 1st the merchants of Belleville are all participating in a huge sidewalk sale. Your group has decided to do all its shopping on this day because of all the special sales.

K-Mart has 24 oz. bags of potato chips, pretzels and taco chips on sale 2 bags for $1.00.

Schnucks has 48 oz. bags of potato chips and pretzels for $1.29. Their taco chips are on special for $0.99 per 48 oz. bag.

Wal Mart has their 12 oz. packages of potato chips, pretzels, and taco chips on sale 3 for $1.00.

National has a special on all their 72 oz. bags of chips, pretzels and taco chips. If you buy one for $1.39 you get one free.

Where would you get your chips and how much of each would you buy? How much would they cost?

CUTTINGS

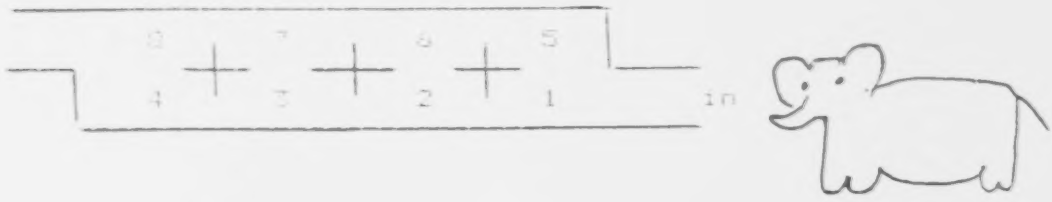
Teachers who come to the Teacher's Center turn in a weekly diary. In it they've written about what they've been reading, activities they've tried with kids and their observations of children's thinking along with general comments they wish to make about what they've been thinking, reading, doing and observing.

Many of these diaries contain material that is too good to remain buried in someone's file; it begs to be shared. So, in "Cuttings" we've simply lifted parts of individual diaries and reprinted them. Read them, think about them,. share them and help them grow.

CUTTINGS
Diane Wooters

As an assignment for Dr. O'Brien's class, I presented the "Elephant and the Biscuits" task to my sixth grade field assignment class:

Find how many different paths the elephant can take to eat a different number of biscuits each day. There is no backtracking. There are no hallways, only doors between rooms. Once he enters a room he eats all the biscuits.



I used 8 groups of students working in groups of two or three for this activity. They had a lot of questions initially, but once they understood the task, they got busy. I was given about 25 minutes out of the regular Math class to do the activity.

I noticed that all of the groups except one added the rooms together to get sums. This one group just used patterns of numbers and no sums. They started using a systematic approach and then the possible paths beginning with 1, 2, 6, but also threw in 1, 5, 6, 2, 3, 7, 4, 8, which is no good because it goes through a wall.

All of the other groups seemed to choose paths randomly, beginning with a 1, 5, 6 path and then writing down a 1, 2, 6 path. Most of the students wrote the paths in this manner: 1, (comma) 5, (comma) 6, (comma) etc. Two groups used plus signs between the room numbers and an equal sign before the sum. Only one group actually showed an addition problem like this:

$$\begin{array}{r} 12 \\ +15 \\ \hline 27 \end{array}$$ to get a sum when the sums got too big.

All of the groups except two traced paths on the ditto, one path right over the previous one. I thought this would be rather confusing but somehow they could see which lines belonged to which path.

Three groups erased the path each time they had drawn it. Most of the groups went up from room #1 rather than sideways.

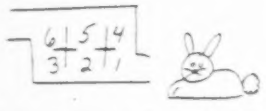
There were two different Math groups mixed together for the activity. They seemed to really enjoy the task once they got into it. All were surprised initially to find out that there were 8 possible solutions.

Overall, the students did very well. They found a system that worked for them. (a system to keep track of their answers, not necessarily one to systematically find the answers)

None of the students thought of making a tree or of finding sums first and then looking for what could be eliminated to get that sum.

Just for fun I had my 7 year old son do the problem. Instead of elephants and biscuits we used a rabbit and carrots. At first Mike seemed frustrated with the rules of the game, but then I suggested that he keep track of days and the number of carrots the rabbit ate for that day. With this start, he happily worked and came up with four possible solutions in a few minutes. I noticed that Mike, just like the sixth graders, started the first move up and not sideways. He felt confident that he had found all the possible paths.

Mike Age 7



DAYS 1 = 1+5 2 36 = 21
2 = 1 2 5 6 = 14
3 = 1 4 5 6 = 16
4 = 1 2 3 6 = 12

CUTTINGS
Barbara Madden

The aims of teaching mathematics are to free students to think for themselves, to provide opportunities for discovery, and to train students in necessary skills. It appears to me, from my own teaching experience and association with fellow teachers, that focus has been on the third aim, with mere lip service to the first and, in general, ignoring altogether the second. Most teachers appear to organize and package mathematics, and present it to the student. I think this is borne out by the fact of recent emphasis by the National Council of Teachers of Mathematics on the need to teach problem solving strategies. One can only wonder why, in most instances, the teaching of mathematics has remained static despite research on how children learn. I would guess that the political systems governing our public schools, the principal in the role of manager rather than instructional leader, and the teacher training programs themselves all play a part. Inertia is a powerful force. But what a wonderfully dynamic and creative place a classroom could be with active learning.

Children should enjoy mathematics. The teacher's enthusiasm must be rekindled if the student is to enjoy the class. I had a good experience a few weeks ago. I had the class make scale drawings of certain objects in the classroom as an exercise in proportion. As the bell rang, one boy remarked, "Class was fun today!"

To tie together this rather rambling comment, I would make these points:

1. School boards and administrators must base their decisions on children's needs, rather than solely on the red and black ink on the ledger sheet.
2. Schedules must be arranged so that blocks of time are available for active learning.
3. Materials must be provided.
4. Teachers must be trained in preservice and inservice programs.
5. Principals must be informed and be a positive force.

CUTTINGS

Beth Jennings

From: **THE MASTER TEACHER**, February 1971

I received this article from my Elementary Methods class this summer and I thought it was worth sharing.

.....The studies reveal learners attain 10% of what they read, 20% of what they hear, 30% of what they see, 50% of what they see and hear. The research proves that learners retain 70% of "what they say as they talk."

However, the most significant finding reveals that learners retain 90% of "what they say as they do a thing." .....The Master Teacher knows that too often teachers "think" students are learning simply because they are quiet and attentive in class.

CUTTINGS
Linda Kahley

Based Upon: **TEACHING MATHEMATICS, 7-13 SLOW LEARNING AND ABLE PUPILS** By Edith Biggs, (Chaps. 1 - 5)

This research seems to have been done in a most efficient and organized and yet informal setting. One gets the impression that Ms. Biggs was conducting remedial math classes rather than research. I guess that would lead to the conclusion that any and all work one does with children can be incorporated into research, if the teacher has her mind and eyes open to what the children are doing rather than focused on what the teacher is doing.

I also wondered, as I read this, why there are no "remedial" or "corrective" math classes offered in public schools. The children who need help with reading are frequently sent to a "resource teacher" for special help, using different types of instructional techniques and smaller groups of children. The children are generally diagnosed and instruction strategies are aimed at the individual child's weak area. With math, however, the children are often grouped by ability, but they receive the same type of instruction just given at a slower pace. There is no diagnosis or individual strategy involved. Why is it that we assume - or expect - all children to be able to learn by the same method of instruction, and at approximately the same pace or they are considered to be "slow" or "backward"?

CUTTINGS
Steve Suess

If ever a book fascinated as well as angered me, **EDUCATION AND THE CULT OF EFFICIENCY** by Raymond E Callahan, is it. Incredulous best describes its impact on 20th Century American education.

At the onset I became bored; it seemed that every chapter simply gave more historical data and background regarding scientific management. I felt as though Callahan was drilling the scientific management theories and practices into my head. Then, *EUREKA*! The analogy between how he was revealing information via the text and what was happening in American education in the first quarter of the 20th century virtually leaped out at me.

With renewed interest I eagerly read on and finished the book, and with each page I realized why we are where we are in U.S. schools today. (Too bad Jean Piaget wasn't more prominent than Frederick Taylor about six decades ago!)

This book should be required for all prospective administrators. It is, at the least, a key to redirecting American education toward a system that is child-oriented, not cost efficient.

Hopefully, intelligent educators will realize that concepts and practices which seem successful in business and/or industry will probably be catastrophic in the classroom.

Saving money by overloading teachers and watering down curriculum is actually quite costly in the long run, and the cost is in terms of lives . . . not dollars.

CUTTINGS
Linda Kahley

Based Upon: **CHILDREN'S IDEAS ABOUT WRITTEN NUMBER**
By Mieko Kamil
Printed in **SEEDBED**

It seems like mathematics is a game with constantly shifting rules: sometimes numbers mean a position; sometimes a set of things; sometimes they're read from left to right (61); and sometimes from right to left (16); sometimes "0" means nothing and sometimes it is extremely significant. Numbers can mean totally different amounts depending upon where they're placed: in 521, the 5 means five hundred; in 51, the 5 means 50; in 15, the 5 means 5 units. The young school age child is just maturing enough to know the concepts of conservation and now the rules don't apply in this new situation.

The language of mathematics is "foreign" as well: one now talks of borrowing, trading and carrying as if the numbers can no longer stand on their own - and why not? - they always have before. The child has to switch his "number concept vocabulary" from numeral as an identifier, (I am 4.) to numeral as indicator of position, (the fourth chair), to a quantitative term (4 chairs), to a combinational factor, (14). That requires so many "gear changes" it's no wonder that some children are lost along the way. When one considers that the beginnings of these concepts are introduced about the same time as the letters of the alphabet are introduced, it becomes an even more staggering task for the child. Letters of the alphabet are confusing in themselves, so now the child is forced to learn two "foreign languages," each with shifting rules, and to learn them simultaneously.

Research indicates the strategies that children employ in trying to make sense of all this confusion. They try to apply reading rules to math and then spell out numbers, 10026, as one hundred twenty-six, and that is an incorrect but ingenious strategy in attempting to make sense of all this new information. Perhaps the strategy building is of greater importance than the error made. Can one correct the error without halting the strategy building and testing?

The fact that children are able to finally make sense of all this speaks volumes about the abilities of children. Why then do many adults view children as not capable of any but the simplest of tasks?

CUTTINGS

Lisa M. Laing

In **CHILDREN'S ARITHMETIC: HOW THEY LEARN IT AND HOW YOU TEACH IT**, written by Herbert Ginsburg, he discusses the fact that young children have a difficult time understanding that one "number word" can represent a group of objects. When asked, these children find it necessary to recount the group's objects, still not finalizing with one number to represent the whole. I saw this happen with 3-year-old Jeremy. When I placed 4 leaves in front of him, he was able to count them, "1, 2, 3, 4", but when I asked him how many there were all together, his answer was "1, 2, 3, 4". (He repeated this when I asked him the same question once again.) Jeremy's brother, T.J., age 5 1/2, was posed with the same question. He counted the four leaves, "1, 2, 3, 4" and then said, "there are four leaves total," in response to the initial question, "how many leaves are there?" T.J. was able to understand that the "number word", 4, could represent the group of 4 objects.

CUTTINGS
Corolyn G. Elden

Why is our present education so far off track. Why are children in so many classrooms given pages of dittos and expected to sit quietly and do them. These things go against the very nature of children. This is why, in my opinion, our youth are in such a state of disarray. We have taught them to be something they are not, and now they do not know who they are. We need to allow children to be who they are, not to teach them to conform to society's norms.

CUTTINGS
Jenice Anderson

H. Whitney has a one point goal for the present day math students. His goal is to help children have a more meaningful time in school.

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SEEDBED

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INTRODUCTION

Although technology has once again reared its ugly head to make the delivery a difficult one (transporting text from an Apple][E to a Macintosh causes headaches), we are proud and happy to bring you two new issues of **SEEDBED.**

SEEDBED 28 owes its being to Professor Thomas C. O'Brien and Mary Steffen whose ideas and enthusiasm for children and the ways children learn open the eyes of all who pass through their classes. The teacher-authors who present their ideas here have some thoughtful, down-to-earth (but not trivial) comments on children's educational growth and well being.

My special thanks to the many "techy" people who worked on our computer problems and at long last found a solution. They showed us problem solving in the real world. Thanks too must go to Laurie Nelson, word processor extrordinaire, and to Angie Fuller, who was always ready to pitch in when needed.

The cover art comes to us from a classroom window art display Teachers' Center travellers saw while visiting classes near Sheffield, England.

Read, enjoy, and continue to share the seeds of learning with your colleagues.

Barbara Goldenhersh
Editor

P.S. GOOD NEWS!! **SEEDBED** is now available on microfiche through Educational Resources Information Center (**ERIC**), Columbus, Ohio.

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DEVELOPMENT OF A MIND ... So That's What Schools Are About!
Shirley Gasparich

Comments On: **Some Thoughts About Rolls-Royces & Treasure-Keeping**, by Dr. Thomas O'Brien

Transmission of information versus the development of a mind ... the two concepts seem so very different from each other that one would hesitate to say them in the same breath. One seems to conjure up an image of "stuff" being moved from one spot to another without changing its shape, form, or content. The other, the development of a mind, seems to be so much more important. It is dynamic, alive, growing, constantly changing, never the same to any two people and never the same to any one person for long. If educators were asked to choose which of the two activities they would want to be involved in, I can't imagine them choosing transmission over the development of a mind. However, all too often, I feel we teachers lose sight of the bigger picture - the larger goal - in our struggle to "get through the material" on schedule.

Perhaps development of the mind is even segmenting the larger whole, the development of the person. Sometimes when a student is particularly bright, we teachers are so eager to challenge, stimulate, and develop his intellect, that we often inadvertently forget about the importance of his growth in other areas as well. In our zest to promote intellectual growth, we often forget that the student has a whole range of emotions and feelings helping or hindering his thought processes on any particular day or at any given time in his life.

In a third grade class in which I'm observing, one "gifted student" spends much of his time daydreaming of his father overseas; he won't see him for another year. Another child's parents are involved in a child custody suit. The child is now living with one parent, but will soon move to another city, changing schools, to live with the other one. Still another child needs ear surgery. He speaks with a speech impediment; he can't understand what's being said unless he is spoken to directly, so he can read the person's lips. Oral directions and spelling quizzes are a nightmare for him, to say nothing of his lack of school success in many other areas. Other students live in single parent families where one parent is trying to provide financially, socially, and emotionally to several youngsters. The list could go on and on.

Then, there is the teacher ... caring, knowledgeable, understanding, and overworked! She seems to be tackling an almost impossible task. Yet ... grades are due, so she must give a math, spelling, and English test.

THE TEACHING OF YOUNG CHILDREN: Book Review

Oleta Kartmann

THE TEACHING OF YOUNG CHILDREN, by Molly Brearley and her six co-authors provides valuable insight into how young children learn and how they can be taught. The organization of the book is helpful in that the various subject areas are reviewed separately. The reader can read as much as pertains to his interests and still enjoy and comprehend the theme of the book.

Overall, the author makes clear her adherence to Piagetian principles of the development of children's intellect. She believes that children can, and will, learn exceptionally well if space, materials and appropriate adult intervention are provided. Regimentation, authoritarian discipline and rote memorization have very little place in her examples of educational theory and practice.

In the chapter, "Mathematics: Ordering, Relating, Measuring," Brearley defines "concrete operational" and "pre-operational" thinking. Pre-operational thinking is defined as being tied to "discrete perceptions" with fluctuations in thinking occurring as perception shifts from one point of concentration to another. In contrast, "concrete operational" thinking can hold a "conserved" spatial relationship with a "reversibility of thought" from beginning to end - backward, forward - while holding both points in a constant relationship. Appropriate examples of each level of thinking are included which further clarify these Piagetian principles.

For the teaching of math Brearley advocates the classroom use of a wide range of "ordinary" materials from the environment with no need of "special apparatus." "Real learning" results through interaction with the environment and thoughtful intervention by a teacher who can take advantage of a given situation to help the child advance his understanding.

In the learning of music concepts, the emphasis is again on exploration and experimentation. A wide variety of musical instruments and objects which can be used to make music should be provided. The children learn to

appreciate music and composition by creating and recording their own patterns of sound. Learning to sing is also taught at first as an experimentation with voice sounds - trying to sing at a certain pitch and to accommodate one's pitch to that of someone else.

The chapter on art is of great interest as Ms. Brearley answers somewhat the question of why kindergarten age children may use paint so much more generously and freely and with much less inhibition than, for example, third graders. She writes that the paintings of young children up to about age five are more related to "sensory-motor exploration than to "symbolic expression." Around age five the child's paintings will begin to have some similarities to real subjects which she terms "analogical expression." Kindergartners are mostly at the sensory-motor exploration level. The analogical expression result in the use of considerable amounts of color and space. As the child matures, his work becomes realistic with more attention given to details. Thus, perhaps, this explains the drawing of houses with paints and brushes; the representations of flowers, birds, sun, and trees of third graders. It does not necessarily explain why fifteen out of twenty-five children in a class would paint virtually the same "house, flowers, etc.

The practice which might explain the stereotyped paintings of eight year olds is one which Molly Brearley gives very little space in her book. She does say that providing students with patterns to draw around, cut out, or fill in is not art, and while it may possibly increase a student's mechanical skill with certain equipment, it does not result in "...the means by which human beings organize their experience of the world in order to further their understanding of it."

The final chapter of the book puts everything together in a classroom setting which shows how the theories, principles, and ideas work in actual practice. Although this book is one which requires considerable concentration by the average reader, it is well worth the time to anyone who is interested in understanding the education of young children.

CAN A THIRD GRADER WRITE A STORY?

Sue Nash

I was called to substitute for two weeks for a third grade teacher. Her lesson plans for the first day was to write a descriptive paragraph using the material they had studied previously. In addition, they were to verbally make up a story about a picture of "The Three Bears", changing some of the story as we went along.

Being a novice at this (I am a former music teacher) I decided to try something different. We got into groups no larger than four, and cooperatively create a fairy tale of our own. If the students used a known fairy tale they had to make it into a parody; otherwise, they could make up a completely original fairy tale. They had to keep in mind that they were to have a beginning, middle, and end. Their sentences had to describe the characters and what was happening so the person reading their story would understand what they were telling.

We also decided to make a book cover and include pictures within our book to help interpret the story for others. Lastly, we set the deadline for Friday to have each group, either act out their story or have a reader stand up and read it to the class.

I was amazed at the excitement this project generated among the students. Instead of the fierce competitiveness usually seen, the groups were interested in entertaining one another and forgot about grades and one-upmanship.

Each group became thoroughly engrossed in their story plot and, as an observer, I saw much interaction of ideas taking place. Did they remember that this was an English assignment? I really doubt it!

A few of the children would leave during English time to go to the Special Education Resource room. The teacher was very cooperative and had the children tell their story to her and she wrote it down exactly as they told it. She then had the children copy the story for themselves. They knew it was their story and were so happy about the results.

Friday came and the children were buzzing with excitement. We moved the desks to one end of the room and set up chairs for the audience. The first children to present their fairy tale were the ones who go to the Special Education Teacher. They invited their teacher to come and watch. Their fairy tale was about living in a nice home in Chicago and having a loving mother and father who picked up their babies and held them tight, kissing them and saying sweet things to them. They did an excellent job in their memory work and in their acting, with props, too! Their Special Education teacher was moved to tears. For these children, their story was a fairy tale, but in it they found a joy of creation in saying, "this is my thought and the way I saw this and there is nothing wrong with it." The other children were very appreciative and applauded with gusto.

From there we proceeded throughout all the fairy tale presentations. Many were truly well thought out and planned, down to many props. Others were shabbily done in the context of our general outline of requirements which we had posted on the board. But mostly, these were students who had not handled other areas with maturity either.

Overall, I felt it was a very successful venture. Of course, one must be ready for some noisier days with groups of four working on creating ideas. But it was well worth it for me -- and for them.

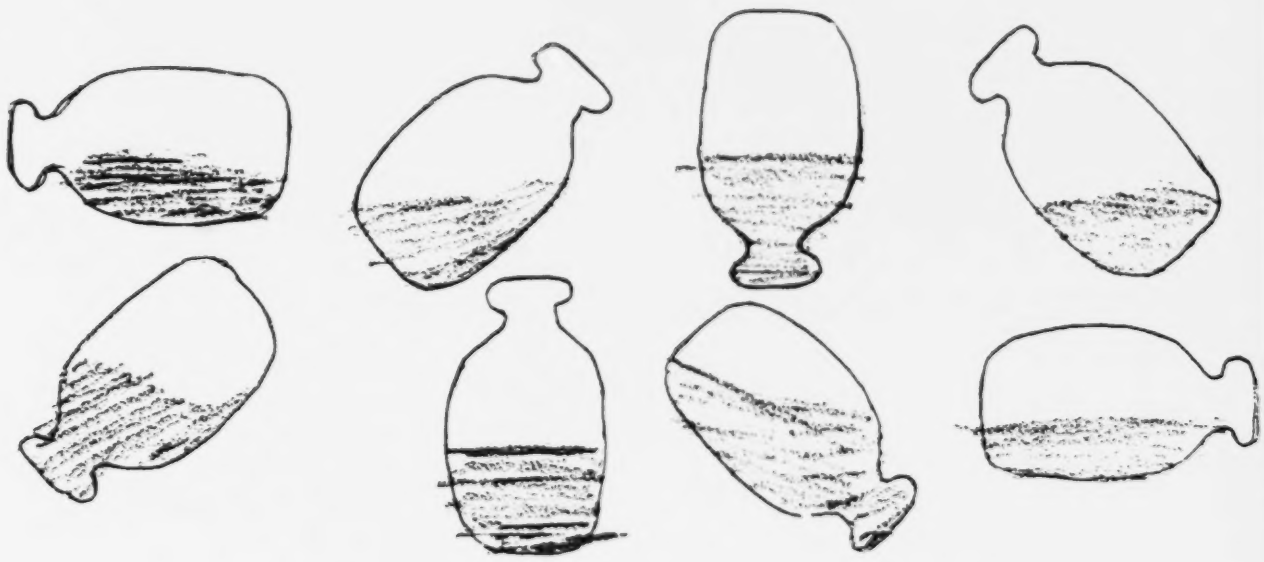
WATER DEFIES GRAVITY?

Lisa M. Laing

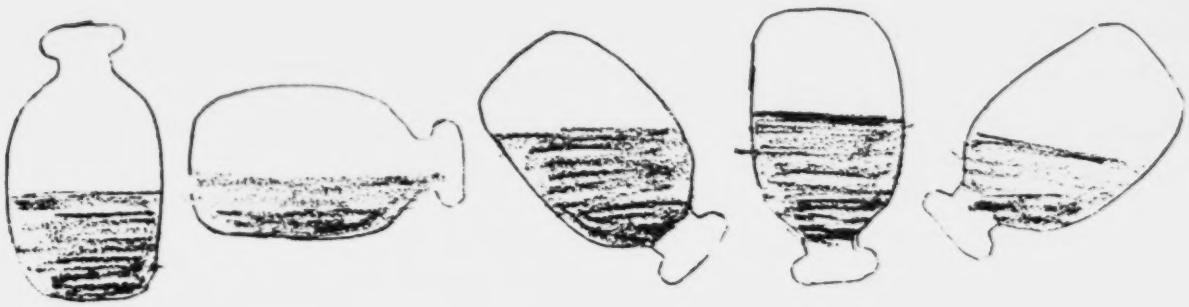
After reading **THE LEARNING THEORY OF PIAGET & INHELDER**, written by Jeanette McCarthy Gallagher and D. Kim Reid, I was anxious to "try out" some of their experiments. One experiment, in particular, caught my attention: Given a clear bottle filled about 1/3 with colored water, would a child be able to see the position of the water and reproduce it on paper, even when the bottle is tilted in several positions. I wanted to try this, and, in addition, see if the child could predict the position of the water when the bottle is tilted.

I knew of four young gentlemen who enjoy helping me with my "college homework." Corey, age 8, was the first to help me with this experiment. I had the bottle with the colored water inside it sitting on a table along with a sheet of paper with eight outlines of the bottle drawn on it in different positions, and a crayon. I was called out of the room for a moment, and in that brief moment, Corey took the crayon and performed part of the experiment with the correct outcomes. When I joined him again, I removed the bottle and asked him to try to "predict" how the position of the water in the bottle will look in several positions. His predictions were correct.

Predictions:



Ryan, age 7, was my next helper. He was able to represent the position of the water very well when he could see the bottle:

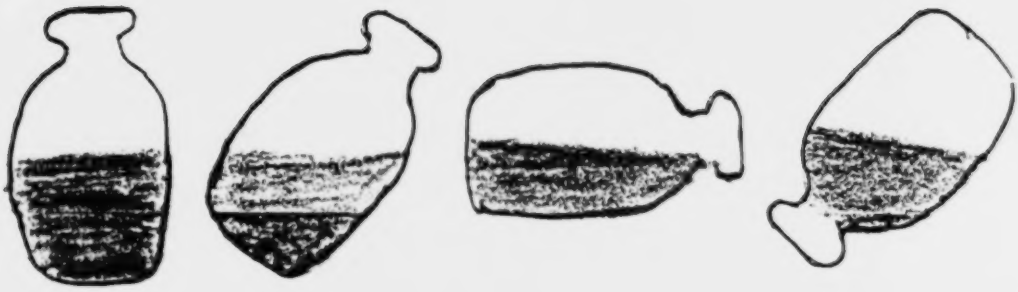


but he had a difficult time predicting outcomes. This part of the experiment was extremely interesting because after he made his predictions, I let him "check" his answers by maneuvering the bottle and making comparisons between the water positions and his predictions. Even when he could see the water in the bottle, he still found it difficult to represent the water positions in the bottle:

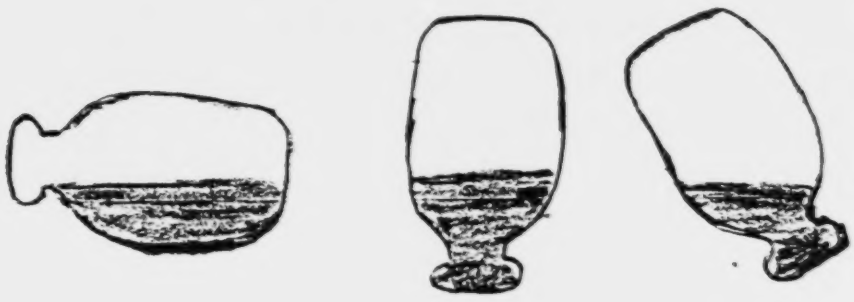
<---This was Ryan's prediction, with the bottle in this position, which he did not change even after seeing the bottle.

Another interesting aspect of the experiment with Ryan was that Ryan never represented the water that was in the "neck" of the bottle in his pictures. He was very consistent in this respect. I am not sure why he made an effort not to include the neck in his drawings. When I asked him if he thought he drew all of the water in the pictures, he said, "Yep, I got it all."

T.J., age 5, was my next "homework helper" as he liked to call himself in these situations. He was also very proficient at representing the water in the bottle when the bottle was present:



and in all situations, with one exception, T.J.'s predictions were correct. (T.J. also did not represent the water in the neck of the bottle.)



(Here, the water in the neck of the bottle was represented.)

EXCEPTION:

prediction

T.J.'s correction after seeing the bottle and "checking" his answer.

Finally, my very patient helper, Jeremy, provided me with an opportunity to see a 3-year-old's mind hard at work putting a task in a perspective that was manageable to him. I showed him the bottle with the colored water inside it. Together, we turned it upside down, sideways, etc. to watch the water "do funny things," as Jeremy phrased it. I presented him with this drawing:

and he showed me how to hold the bottle so that they were the "same". (This concept of "sameness" appeared to be easy for Jeremy to understand.) Then, I asked him if he could point to the area on the bottle where the water was. He did this without any hesitation at all, but when I asked him if he could color the water in his picture so that it would look just like the "real" bottle in front of him, he just nodded and proceeded to represent the water in the bottle:

We went through the same process (with some variations in how I asked him to represent the water) five more times with the same results. Jeremy helped me to understand that he could see different positions of the water in the bottle, but he still had a difficult time representing what he saw.

REFLECTIONS ON THE "BRITISH SEEDBED"

Elaine Edwards

I was very interested in Seedbed 24. I was so impressed by the ideas and atmosphere of the British schools described in these articles that I wish I were school-aged in Britain. I hope my children are lucky enough to experience a British style American classroom before they are too far along in their schooling.

I am totally in agreement with the idea of the integrated curriculum. In my opinion, it's an approach in which knowledge is more easily assimilated into the child's "world view." I have always believed that the best education does not teach facts (or even subjects) in isolation, but always shows how it relates to the Big Picture, how it relates to the real world, where it fits in the scheme of things.

Interestingly, the author says the greatest advantage of the integrated curriculum is the students' feeling of ownership. It would be easier to embrace ideas that had been explored from many angles until their larger meanings became evident. Compare that to the study of isolated, arbitrary facts. The general reaction becomes, "Who cares?"

I was also very impressed by Britain's position of "head teacher." Since this is a master teacher, coming up through the teaching ranks; he or she would be more likely to hold the respect of the other teachers. Keeping in touch with the classroom as he or she takes over for teachers in actual classrooms, would also

keep him aware of the day-to-day realities of teaching.

Having been a travelling teacher, I've run into my fair share of pompous, authoritarian bureaucrats. Since Britain's head teacher sounds so ideal, it makes me curious about the structure of their educational system. Does the head teacher have to walk the three-way tightrope between teachers, parents, and the (sometimes woefully inadequate) school board? Or do they have a little more security, and thus can afford to be more relaxed and truly helpful.

Another thought -- someone who is more accessible (and a good example!) as the head teacher, is more likely to inspire teachers and bring out their best performance. If the teachers have to be "guarded" in the presence of the principal (he evaluates your teaching and decides if you get tenure) you are less likely to bring a problem to him for fear of being thought a poor teacher.

I was also impressed by the absence of constant testing and grading to mark pupils' progress. When students aren't constantly being evaluated (or graded) they may feel freer to ask "dumb questions" and find the answer or solve the problem for their own satisfaction.

I really liked the emphasis on writing out all solutions and how they were arrived at. This is so much more motivating than just getting graded on whether you got **THE** one correct answer, with no regard for the problem-solving process used.

WHAT CAN I DO?
Steven Suess

As a father and an educator I am concerned about the future of American education especially in the elementary schools. What can I do to insure my 5-year-old daughter develops into an intelligent and independent individual capable of critical thinking?

First, I can become familiar with the curriculum and philosophy of the school she attends. It is not enough, however, to pursue goals and objectives that seem excellent; instead, one must ask the teachers and the administration how they plan to implement such goals and objectives. Quite simply, by what means are students going to be urged to be problem solvers.

Next, I can provide a home environment that challenges independent thinking, motivates trial and error tasks, and develops positive self worth. How? Well, two ways come to mind. The first is by my example as a human being dealing with everyday situations. The second is by supplying games, activities, and opportunities for my child that are geared for the development I desire and facilitating such.

I realize that my suggestions are time consuming, but can you think of a more valuable investment than the education of your child?

A DELAYED LESSON

Laura Hoeke

As a child, I remember setting off on my paper route and counting as I walked. For several weeks I remember having a challenge with myself. I wanted to see if I could reach a higher number each day. I did this until I reached numbers that were confusing to me -- for example what came after 1,999; was it 2,000 or 10,000. At that point, I felt silly not knowing the answer and did not have the guts to ask such a dumb question. So, I stopped trying to count higher and eventually grew into the knowledge I lacked.

Now, I am surprised and somewhat relieved to discover that many children of that same age (I was 12 or 13 at the time) have the same or similar questions about counting, and probably experience the same frustration with finding an answer. This was brought to my attention through an observation task. I was to ask children to count from a number until they were asked to stop. Here's the list of numbers used.

35 - 46

93 - 108 - 118

193 - 208

993 - 1004 - 1014

10,993 - 11,004

I found it common for children through 6th grade (the oldest I worked with) have difficulty with the counting task. They would often pause, skip or repeat numbers before transitions, (i.e. from 99 - 100). A few students simply refused to go any further for fear of giving the wrong answer.

This whole episode once again piqued my own interest in counting. Although I had long ago learned how to count up to a million, I was somewhat befuddled by the numbers beyond. During my younger years I had come upon the crazy conclusion that to reach a billion, I not only had to reach nine hundred ninety-nine million. I had to fit the thousands somewhere above that. So, my highest million was nine hundred ninety nine thousand million, nine hundred ninety nine million, nine hundred ninety nine thousand, nine hundred ninety nine. (I wrote the number long hand because as I was working through this discovery I never did actually write out the number. Had I, I could never had denied the fact that I was actually working in the billions and I just was not ready to show myself conclusively that my invention was wrong.) I spent quite some time during the past few months trying to rearrange the numeration system to fit my invention. Finally, I reached the point that no longer allowed me to retain my delusions. I had to face up to the fact that long ago someone decided that the numeration system would be based on ones, tens, and hundreds, and used those building blocks to complete each of the following levels; thousands, millions, etc. My mistake had been in thinking that I had to go through all the preceding levels in order to reach the next level.

I spent a lot of years trying to avoid figuring out a simple problem because I thought it was stupid, or worse yet that I was stupid for not understanding. To tell you the truth many of my friends think I've flipped my wig for trying to figure it out. "It's so easy!" they say. I've got to agree with them it is easy, but I don't feel stupid for having been 22 when I figured it out. I'm downright proud of the fact. At least now, I understand.

DAILY PROBLEM SOLVING
Beth Jennings

Children can see relationships between objects they encounter in their daily lives if a teacher learns to use every precious moment as a learning experience. Often one is so involved in getting in all the "curriculum" that some of the best learning experiences are lost.

Children will pattern, sort, and classify objects and then become interested in comparing these groups. These comparisons can be used as graphs to see more and less, how many more or less, and to put the information in order.

I would suggest that any teacher of young children obtain a copy of **MATHEMATICS THEIR WAY** by Mary Baratta-Lorton (Addison-Wesley). It is an excellent book giving ideas of concrete ways to accomplish patterning, sorting, classifying, comparing, graphing, and number introduction. As I read the ideas in the book, other examples came to mind of activities that could be performed with materials readily available in a classroom.

The most important points are:

TAKE ADVANTAGE OF DAILY SITUATIONS

and

DO NOT BE AFRAID TO INCORPORATE EVERY MOMENT INTO A MEANINGFUL LEARNING EXPERIENCE.

Children learn faster and retain the learning if they are interested and actively involved in situations.

EXAMPLES OF DAILY PROBLEM SOLVING ACTIVITIES

ATTENDANCE - number of boys and girls absent, present, total
LUNCH COUNT - buying, bringing
TRANSPORTATION TO SCHOOL - bus, walk, car
PASSING OUT PAPERS, SNACKS , MATERIALS
PATTERNING - reading clothes, calendar patterns, rhythmic patterns, children patterns - boy, girl; standing, sitting; etc. patterns with materials or objects

SORTING - buttons, cars, attributes - size, shape, color, texture
GRAPHING - This is an area that really interests me and I will go into detail.

GRAPHING

Graphing is a problem solving tool used to help children see relationships.

1. REAL GRAPHS - compare groups of real objects.

EXAMPLES: shoes/boots, vegetables, colored leaves, mittens/gloves, cars/trucks, plastic animals, etc. A floor graph can be made by using tape to mark off grids on a piece of vinyl. Grids could also be laminated on poster board to use for smaller items on a table.

2. PICTURE GRAPHS - use pictures or models to stand for real objects.

EXAMPLES: bus, car, child, happy or sad face

3. SYMBOLIC GRAPHS - symbols stand for real things Xeroxed pictures can be made of each child in the c lass and used when children make a graph of their own behavior.

EXAMPLES: Are you going trick or treating? Do you have an artificial or real Christmas tree? Would you like to -- or --?

Real graphs form a foundation for all graphing activities. Situations can be repeated using all three forms of graphs. Students can discuss and make real and picture graphs permanent by recording them on a symbolic graph.

OTHER EXAMPLES:

Weather,	Favorite (anything),
Lost teeth,	Feelings,
Temperature,	Birthdays,
Yes / no questions.	

CONSERVATION OF VOLUME

Barb Madden

I have read that conservation of volume is one of the last conservation skills to be developed and is still unstable with some junior high students. This week I had the following experience with my fifth hour class of eighth graders.

We were studying the volume of prisms. One boy asked a question about the volume of a triangular prism. He knew the proper formula:

$V = B \times h$.

I asked him which side is the base. He hesitated and appeared confused.

We began to examine the prism. There were two triangles and three parallelograms. We rested it on its various faces. I asked, "Does the volume remain the same when we place it in these various positions?"

The class responded with a unanimous, "Yes!"

But they were still uncertain as to which side was the base. Finally, some reasoned that if the base is the bottom, the base must be the side it's resting on.

I restated their hypothesis. "If the prism is resting on the parallelogram, the parallelogram is the base; if it is resting on the triangle, the triangle is the base."

"Yes!"
"Does the volume remain the same?"
"Yes!"

I then asked how to apply the rule: $V = B \times h$. One boy suggested, and the class agreed, that we use $B = b \times h$ if the prism is resting on a parallelogram and $B = 1/2 \times b \times h$ if resting on a triangle.

I asked, "Will this work?
"Yes!"
"How do you know?"
"We'll try it!"

The class took the proper dimensions of the prism. Half the class solved the problem using one base; the remainder used the other base. They were astonished to find that one answer was twice the other.

"Which one do we use?" I asked.

They were puzzled over this. One student gave the remarkable response, "If it is resting on the parallelogram use that answer (he pointed to the larger one), and if resting on the triangle, use the other one." No one disagreed.

But I was astonished to hear them say that position was a factor of volume. This was a direct contradiction of their earlier statement that the volume remains the same regardless of position.

I focused their attention away from the numerical answer and back to the prism itself. I again asked the question, "If I lay the prism on the parallelogram, it can hold the larger amount, and if I place it on the triangular base it holds the smaller amount?"

"Yes!"

There was quiet. Then a few students made some feeble attempts to explain this. Finally, one girl said, "Wait, the volume must stay the same!"

Her comment seemed to pull the students back to reality. The volume had to remain the same. But which answer was the right one?

The students thought again. Then one boy recalled that we had found the volume of a triangular prism by slicing a rectangular prism along the diagonal. He realized that the smaller value must be correct. We then returned to our original discussion of which side is the base.

This experience points out to me that I should make no assumptions about the thinking of my students.

HOW MANY 7'S?

Deborah Beasley

How many 7's would it take to print a grid of numbers from 1 to 100? 1 to 1,000? 1 to 10,000?

1 to 100

I simply wrote every number that had a 7 in it between 1 and 100. Then I added the number 7's.

7, 17, 27, 37, 47, 57, 67,
70, 71, 72, 73, 74, 75, 76
77, 78, 79, 87, 97 20

1 to 1,000

I know that there are twenty sevens in every 1 to 100 group of numbers, except the 700's which have an extra 100 7's due to the 7 in the hundreds place. I wrote each group of hundred and the number of 7's contained in that group, then added the sums.

0 - 99: 20
100 - 199: 20
200 - 299: 20
300 - 399: 20
400 - 499: 20
500 - 599: 20
600 - 699: 20
700 - 799: 120
800 - 899: 20
900 -1,000: 20 300

1 to 10,000

I know there are 300 7's in each 1,000 except the 7,000's which have an extra 1,000 7's due to the 7 in the thousand's place.

300 : in each 1000 except the 7,000's
X 9 : # of thousands in 10,000 except 7000's
2700 :
+1000 : extra 7's due to 7 in the thousand's place
+ 300 : # of 7's in 7,000 other than thousands place
4,000 : total number of 7's to print 1 to 10,000

I recognized a pattern and decided there had to be a formula to represent this pattern.

STEP 1 Pattern is
20 - 7's to print up to 100
300 - 7's to print up to 1,000
4,000 - 7's to print up to 10,000
50,000 - 7's to print up to 100,000

STEP 2 $20 = X$ I was trying to represent the pattern
$(X + 1)10$ algebraically. This attempt didn't
$[(X + 1)10]+1$ work.

I approached the problem from a different angle.

STEP 3 $X = 2$
$10X$ = # of 7's to print up to 100
$(X + 1)102$ = # of 7's to print up to 1,000
$(X + 2)103$ = # of 7's to print up to 10,000
$(X + 3)104$ = # of 7's to print up to 100,000
$(X + 4)105$ = # of 7's to print up to 1,000,000

This step didn't work either.

STEP 4 I left the algebra for a moment to look at the visual pattern of

the numbers.

in 100 there are 2 - 0's 2 places 20
in 1,000 there are 3 - 0's 3 places 300
in 10,000 there are 4 - 0's 4 places 4,000

STEP 5 This didn't satisfy me so I tried algebra again.
A=10 X=2 N=
AN, AN + 1, AN + Z,.....

STEP 6 I was trying to represent the pattern.
# of 7's in 100 = 10X
# of 7's in 1002 = (10X)2

I made a mistake in step 6 which I later caught:

1002 = 10,000
(10X)2 = 4,000

STEP 7 There were 4,000 7's up to 1,000 not 10,000
if in 100 there are 20 - 7's and
in 1,000 there are 300 - 7's, then in
10,000 there are 4,000 - 7's.

STEP 8 I was representing the pattern again.
X = 2
10X = # of 7's in 100
102(X + 1) = # of 7's in 1002
103(X + 2) = # of 7's in 1003

I realized here that 1002 = 10,000. What I was looking for was 1,000. I realized another way to write 100 is 10 X 102. I found how to represent the pattern.

STEP 9 10(n)[2 + (n - 1)] = # of 7's needed to print from
1 to (10 X 10n)

STEP 10 My last step was to check my formula:

if n=1: 101[2+(1-1)]=# of 7's to print 1 to (10X101)

10[2 + 0] =

10 X 2 =

20

if n=Z: 102[Z+(2-1)]=# of 7's to print 1 to (10X102)

100[2 + 1] =

100 X 3 =

300

if n=3: 103[2+(3-r)]=# of 7's to print 1 to (10+103)

1,000[2 + 2] =

1,000 X 4 =

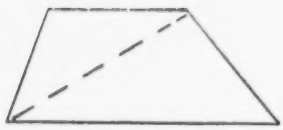
4,000

My formula works!

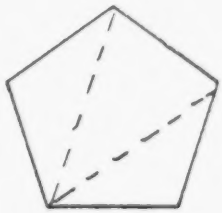
HEY, POLYGON, WHAT'S YOUR ANGLES?

Marilyn Hughes

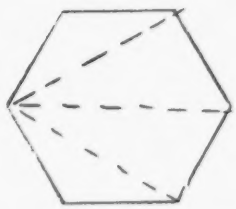
Every geometry course at one time or another will discuss polygons and the sum of the interior angles. The approach usually taken is to divide the polygon into triangles. The student would already know at this point that the sum of the angles of a triangle is 180 degrees. Thus with a little explanation one can convince the student of the formula (n - 2)180, where n represents the number of sides that the polygon has.



2(180) = 360°

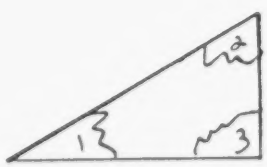


3(180) = 540°

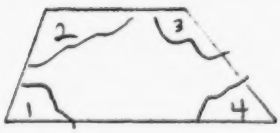


4(180) = 720°

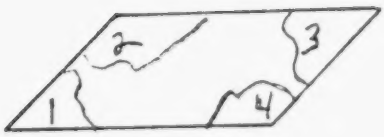
While looking at **THE GOOD TIME MATH EVENT BOOK** by Marilyn Burns, I found another approach to this problem. The students can begin by cutting out several different triangles from construction paper. Next they are asked to tear off the three corners, and paste or tape them down, side by side, so each corner touches the same point. The students can quickly see that the three angles form a straight line, or 180 degrees. They can compare their triangles to others around them. Do the size, shape, angles, yield the same thing?



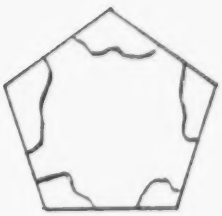
What happens with four sided polygons? What about five sided ones? After students have done lots of polygons and spent sufficient time with the models, they will be able to figure out a rule for all polygons. When they do, it will be their rule and theirs forever.

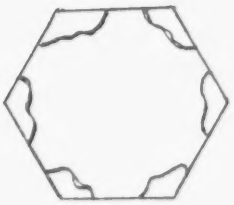


360° ?



360°





EXCERPT

Jerry Pournelle

Reprinted from INFORMATION WORLD: November 10, 1986

(Jerry Pournelle is a noted science fiction writer and computer columnist)

Computers exist and proliferate. They get more powerful even as their prices fall.

I recall a time when many of my friends were concerned that the computer revolution would create an enormous gap between the rich, who could afford the machines, and the poor, who couldn't. I said at the time that was a false argument. Now it's obvious. The Atari ST, a machine far more powerful than any of us owned in the '70s, sells for less than a year's supply of cigarettes.

By the year 2000, anyone in western civilization will be able to get the answer to any question that has an answer.

If you grant that prediction, any education based on knowing facts will be worthless. In that era anyone will be able to get any fact, probably a lot faster than we now suppose. Knowing "facts" won't be worth diddly. What will be important is knowing how to get answers - and more importantly, what questions to ask.

PRESCHOOL LEARNING

Brenda Thomas

Name Recognition:

I teach a four year old preschool class. All the children in my class can spell and recognize their first names. I wanted to challenge the children, so I began writing only the children's last names on their papers. It was their job to locate their own paper. At first, this was a complete mystery to them. I watched them begin to solve this mystery. Some children knew the letters in their name, others knew if their name was long or short, and they all found a method to recognize their own paper. They have not had any help from me, and they can all find their own paper, after only one week. In short, the children enjoyed this much more than if I had made name cards and held them up using a method of constant repetition to learn their names, and they learned faster by having to complete this job on their own. No teaching took place but there was a lot of learning in a short period of time.

Position:

We have a set of cards showing the different positions i.e. over, under, beside, on top, in back, in front, inside, outside. I showed these cards to each child one on one. In many cases they were having difficulty telling the positions to me. However, there was a car on top of a garage, and they all got this one right. I believe children can learn from the absurd. We did another exercise in this same activity one week later, and all the children knew the position on top of, even though they were still having difficulty with the other positions. Since this time, I have created some absurd situations, such as, placing a doll in a cabinet and under a bed. Invariably the children are able to name the correct position for the doll. Perhaps we need to spend less time drilling and more time setting up learning situations.

WHY I STILL TEACH
Frank Barone

Twenty years ago, toward the end of the semester on a sunny June day, I asked John, one of my seniors at Poway High School, if he had learned anything in our class. He looked up from his desk and said, Yes, I have."

I followed up with a second question. "How do you know that you've learned?"

He looked down for a moment, then raised his head, looked me straight in the eyes and said, "Because I've had fun."

I thought about John's answer, then smiled. After thirty years of teaching, I'm still smiling.

Since that time I have used John's profoundly simple statement as one measure for the success of my teaching. Perhaps John didn't have a philosophical basis for his answer. It didn't matter. He answered sincerely, honestly, more from his heart than his head. Later I would search for and discover all the educational philosophy I would need to support the affective atmosphere in my classroom where students like John can enjoy learning in a non-threatening, accepting environment.

I have another measure of success in my classroom. It also comes from students. On several occasions I have asked them to identify the single characteristic of a good teacher. Almost as a group they have answered that a good teacher cares, and not just about subject matter, but he or she cares about them as people, as learners.

Let me tell you about caring. Jan graduated in 1974. I recently received a note from her in which she says, "I've thought about you almost daily (honestly) and I thank God that you were there to "save" me in high school. I will never forget how much I grew behind those blue doors with you.

I like to see students grow. I like to see a classroom of strangers turn into a circle of friends, to see students listening to each other as they share their experiences and writing in our learning circle, and to hear the spontaneous applause that comes when they acknowledge the honest voice behind a well-crafted poem.

I like to see metaphors fluttering from their papers like butterflies emerging from crusted cocoons, then cover the bulletin boards with poems and publish them in magazines so that I can see the names of my "up and coming" young

poets in print. I still delight in seeing students "show not tell" in writing that "sings into the eyes and touches the heart."

Teaching still gives me pleasure when a non-reader picks up a book and crawls under a table to read undisturbed, or when one of my non-writers asks me in a loud voice from across the room, "How do you spell cucumber?" because now spelling has become important to him as he writes his stories.

I have fun teaching because I like to share in the excitement of my students when they bring in their "cute and colorful" creative projects.

I enjoy their enthusiasm when they come with their "Surprises" to share with our class, like the five students who hired two professional clowns to entertain us as their required "Surprise" for the semester.

I want to stay around for another Ten Year Reunion at the Hotel Intercontinental and mingle with the elegantly dressed young ladies and the well-groomed men dressed in their three-piece business suits as they renew teenage friendships and relive high school memories. It makes me feel good when they say, "I am still writing in my Journal."

I experience other satisfying moments in teaching, like sitting down at lunch with Gary and exchanging the paragraphs we wrote for our Twenty-One Senses, or sharing poems and one-act plays with Gerry, our resident playwright. That combination of fun and learning happens every time Sam comes to talk with my students about poetry. And I enjoy making in-service presentations for Linda, Carol, Don, Joyce, Bob, and ... and ... and... .

Early in my career a professor told me, "Why we became teachers is not as important as why we continue to teach." I know why I continue to teach -- because even with crowded classrooms, the heavy paper load, and the difficult third period class we all experience from time to time, I'm still having fun and I still care about young people.

The blue doors behind which Jan grew in 1974 have since been painted orange. But as long as I can continue to make a significant difference in the lives of students like Jan, you will still find this teacher doing what he likes to do best - helping students grow. And I'll grow along with them.

PRESENTING PROBLEM SOLVING CHALLENGES

Virginia Small

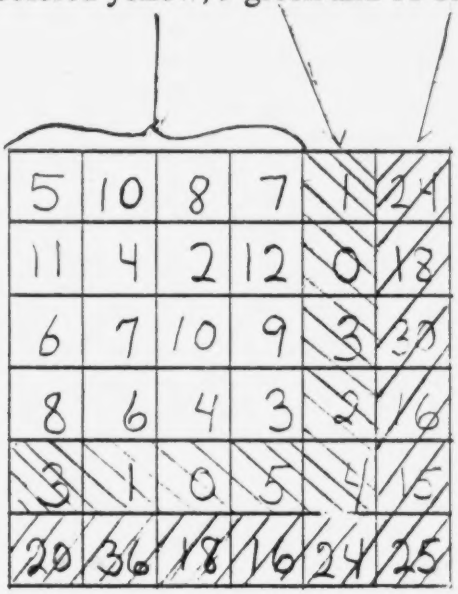
I have no idea what the world will be like in the 21st Century when my first graders will be adults but I can prepare them in many ways. One of the ways is to not be afraid to tackle a problem that at first glance seems too difficult to solve. Most children feel more secure with addition or subtraction problems that are solved without a challenge to their thinking. Real life problems aren't that simple to solve. The idea is to present problems that will challenge them as well as help them not to be afraid to keep trying if success isn't there on the first try. Important also is the idea that there is possibly more than one approach or solution.

I teach our basic curriculum, but along the way I add a problem solving unit with as many different kinds of problem solving approaches as I can find. I always keep searching and trying out new ideas. Many of these ideas can be used at various times of day when there are a few minutes that appear in your schedule when waiting to line up for a program or a specialist. An example is to write LESS THAN and GREATER THAN on the board and tell the children you are thinking of a number between 1 - 15. They are to guess a number. The number they guess is placed under the appropriate heading. For example if I was thinking of number 3 and the child guessed number 7 I would place the 7 under the heading GREATER THAN because 7 is greater than the number 3 I was thinking of. If 2 was guessed it would be placed under the heading LESS THAN.

Have you ever thought of using dice as part of your problem solving unit? I hadn't until I attended our Teacher Center class and heard Jerzy Cwirko-Godycki tell us of many activities you can present with dice. We have all seen and used dice without thinking of classroom uses for problem solving. Ask the children to really examine some dice. What are some of the things they observe? Hopefully some will mention the "dots". How much will the dots on the opposite sides add up to? If you haven't thought about this get some dice and see for yourself. Place 2 dice together and ask if your students can figure out the total of the hidden middle sides. Try using 3 dice and see if the hidden

sides can be totaled.

I recently found a game called GIANT DICE. It has two large dice, one red and one green. There are 4 colorful cards similar to bingo cards with 36 squares on each, 16 colored yellow, 9 green and 11 blue.



5	10	8	7	1	24
11	4	2	12	0	18
6	7	10	9	3	20
8	6	4	3	2	16
9	1	0	5	4	15
20	36	18	16	24	25

The first game is addition and only the yellow squares are used. The dice are rolled, the numbers added and a counter placed over a number on the card. The first player to have 4 counters in a row is the winner. The second game is adding and subtracting and used the yellow and green squares on the card. This game uses more strategy because the player may either add or subtract to place a counter on the desired number on the card. This game requires 5 counter in a row to win. The third game uses the whole card and the dice may be added, subtracted or multiplied. The first player to have six counters in a row wins. The last game uses adding, subtracting, multiplying, and dividing. The game is played as before, but the giant in arithmetic now has a much more difficult extra alternative. The player may add the numbers thrown, then find any two free numbers on the card which give this number when the small is divided into the larger. As a reward the child may cover

both numbers. I have also found these colored dice are great for other games. This game has 60 colorful counters and these can also be used for great thinking games.

Have the child take a handful of counters and count them. It is very interesting to see how this is done. Some move them, some try to count without moving any, some may stack them. Take a few counters and have the child count these and hide a few under your hand and ask the child if he can tell you how many are missing

During the August, 1985 summer session of the Teacher's Center our guest from England, Mr. E. A. Osborn (Ted) talked to us about environmental mathematics. This is a form of math rarely used in our classrooms. We divided into groups and were to observe our environment. We made our observations on an adult level. I am trying to use this same technique with my first graders. We observed the patterns of our brick walls of our school building. We took a small area of the wall to decide if we could estimate how many bricks would be in the area. Estimation is of great value in many life situations but rarely used in classrooms in any part of the curriculum. Symmetry in our environment was another property we observed comparing leaves by folding them in half to see if they were symmetrical. Our Social Studies curriculum shows maps of classrooms. We drew maps of our classroom and observed if the patterns our desks made were symmetrical. We noticed even numbers of desks of symmetrical and an odd number of desks were not.

I am sure as our observations continue we will find many more ways to become aware of the practical uses of math in our environment. A special thank you to the Teacher's Center Project and its many and varied guests who have made the participants much more aware of how children think and the importance of varied approaches to learning about our world.

SOME THOUGHTS ON EDUCATION

Patricia Lally

Although I have not regularly taught in a class of very young children, I have often wondered at which age or stage a child's creativity becomes stifled. How does a child learn to become a "robot", feeding back only the ideas and answers which the teacher has given to him/her? Where does a child learn that it is always necessary to have the "right" answer arrived at in the teacher's way? I like the use of the term "squashing effect"1 because this is exactly what we do to children in the guise of educating them.

By the time students arrived in my junior high history class, I found many who were uninterested, bored, and wanting to do as little as they had to do. Some, however, wanted an "A" for just doing exactly what they were told. Open-ended assignments requiring creativity scared them. Most constantly demanded I tell them what to do and how to do it. A few were at my desk with every step of the assignment: "Is this right?"

I have an answer for this problem which does not seem very practical in our present school structure. All classes could be conducted on a seminar basis, using small groups and limiting class size to 20 students maximum. In earlier schools where a teacher taught all grades, grouping was not only necessary, but flexibility most important. Human resources were used practically, as students learned from helping each other.

I found that it took a lot of time and trouble in my classroom to attempt to

awaken curiosity and to help students feel comfortable with "thinking for themselves." Listed below are a few ideas that may prove useful to others as they did to me.

1. Do not give facial encouragement to right answers only, but make sure that children understand that we learn from our mistakes.

2. Be willing to leave questions unanswered sometimes pending further investigation--also to back up when needed.

3. Give grades in many areas rather than relying only on written tests. Creativity, group interaction, projects which demonstrate thinking skills are important measures of learning.

4. Pause when asking questions to give children time to assimilate, process information, and respond.

5. Give children chances to progress at their own pace.

6. Listen to a child as you expect him/her to listen to you. Give the child your full attention.

7. If you are open, curious, accepting, and interested in new ideas, your students will be.

O'Brien, T. and Whitney, H., **"Can Children Remain Themselves in the Classroom?"**

A PARENT'S PRAYER
Abigail Van Buren
January 11-12, 1969

O Heavenly Father, make me a better parent. Teach me to understand my children, to listen patiently to what they have to say, and to answer all their questions kindly. Keep me from interrupting them or contradicting them. Make me courteous to them as I would have them be to me. Forbid that I should ever laugh at their mistakes, or resort to shame or ridicule when they displease me. May I never punish them for my own selfish satisfaction or to show my power.

Let me not tempt my child to lie or steal. And guide me hour by hour that I may demonstrate by all I say and do that honesty produces happiness.

Reduce, I pray, the meaness in me. And when I am out of sorts, help me, O Lord, to hold my tongue.

May I ever be mindful that my children are children and I should not expect of them the judgment of adults.

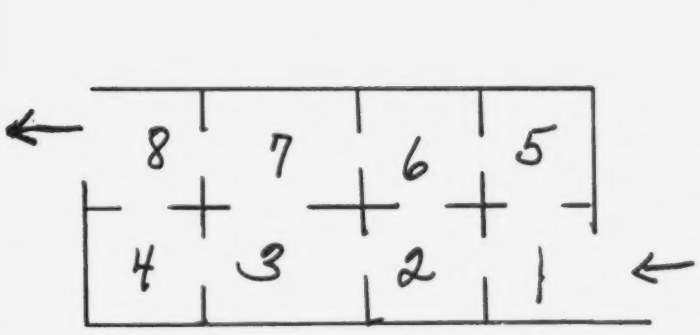
Let me not rob them of the opportunity to wait on themselves and to make decisions.

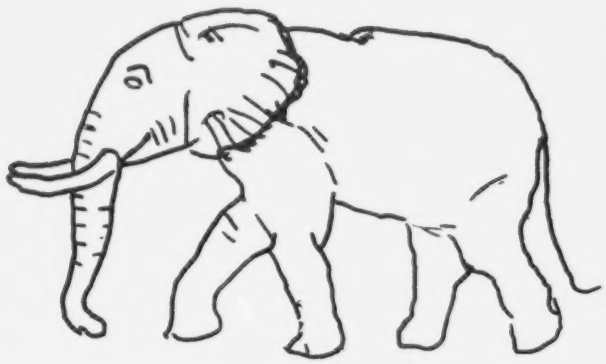
Bless me with the bigness to grant them all their reasonable requests and the courage to deny them privileges I know will do them harm.

Make me fair and just and kind. And fit me, O Lord, to be loved and respected and imitated by my children.

ELEPHANT AND BUNS PROBLEM

Barbara Madden





The problem is to determine the number of buns the hungry elephant can eat as he passes through the house. The numbers represent the number of buns in each room. The elephant cannot backtrack, and, as he passes through each room, he must eat all the buns in that room.

I observed four different groups of eighth grade students.

Darick and Todd: Todd was passive, but Darick aggressively tackled the problem. He quickly traced the paths to find the maximum and minimum numbers -- 18 and 34. For quite awhile he played around with the paths in a rather random way. He wrote the numbers from 18 to 34, and by trial and error began testing each number. This became confusing, and the pair weren't sure they had tried all possible paths.

Soon they abandoned this method. They first traced the paths with 5 numbers and then the paths with seven, and made the following table. They moved toward economy.

1 2 3 4 8	18
1 2 3 7 8	21
1 2 6 7 8	24
1 5 6 7 8	27
1 5 6 2 3 7 8	32
1 5 6 7 3 4 8	34
1 5 6 2 3 4 8	29
1 2 6 7 3 4 8	31

They had organized the problem. They abstracted the problem. It was no longer a problem of elephants and buns, or even of sums. It was a problem of paths. Finding the sum was a trivial computation task.

Darick suggested a better name for the puzzle -- "The Elephant and the Peanuts."

Jenny and Jill: This group had a similar method. They first found the greatest and the least. They centered on paths rather than on sums. This can be seen by the order in which they show the sums, neither increasing nor decreasing. Note also that they used neither "=" nor "+". They made this table for the remaining paths.

1 5 6 7 8	27
1 5 6 2 3 7 8	32
1 2 6 7 8	24
1 2 3 7 8	21
1 5 6 2 3 4 8	29
1 2 6 7 3 4 8	31

Jason and Scott: This group transformed the problem into a "bypass" problem. After experimenting with the problem, they determined that the elephant can bypass one room or three rooms and get through the house. The elephant can bypass no other number or rooms. After finding that the total number of buns in the house was 36, they made this chart.

1 room bypass:	room 2	36 - 2 = 34
	room 7	36 - 7 = 29
	room 5	36 - 5 = 31
	room 4	36 - 4 = 32
3 room bypass	rooms 2, 3, 4	36 - 9 = 27
	rooms 4, 5, 6	36 - 15 = 21
	rooms 5, 6, 7	36 - 18 = 18

They observed that one could never bypass 1 and 8, and that in a three room bypass, the numbers would always be in consecutive order.

Jason and Scott were interested in the problem and pleased with their strategy, so I suggested that they apply it to a ten room house

In the ten room house they observed that since there were an odd number of rooms in each row, one could pass through all the rooms. They also

determined that the number of rooms one could bypass would be either 2 or 4 and nothing else. They again made a table, this time subtracting the total from 55.

Danny, Craig, and Ricky: This group considered the rooms in two parts. They reasoned that the possible ways to exit are 3 -> 4 -> 8 and 7 -> 8 which both give 15, or 3 -> 7 -> 8 which gives 18, or 7 -> 3 -> 4 -> 8 giving 22.

Then they concerned themselves with the remaining four rooms and found all possible paths by joining the two parts of the problem.

```
                    15
1 -> 5 -> 6 -> 2 -> /
                    \18

                  15
1 -> 5 -> 6 -> /
                \22

           15
1 -> 2-> /
         \18

                15
1 -> 2-> 6-> /
              \22
```

SO WHAT?

1. The mind tends to break up a problem into manageable parts, find patterns and relations, and restructure the problem into a new whole. The students deformed the problem, found short cuts, moved toward economy.
2. The students abstracted from the problem. It was not one of elephants or even one of sums, but of paths.
3. There are many ways to solve a problem. Students learn from each other. The teacher does not have all the answers.
4. The students had fun. To solve a problem, to establish equilibrium, is satisfying.

DATES: WHAT DOES MY AUNT'S BIRTHDAY HAVE TO DO WITH PIAGET ?

Oleta Kartmann

Have you ever visited a cemetery and while looking at a gravestone calculated the age of the deceased? Suppose the dates were "B. 1895 - D. 1932." Was your first impulse to subtract just the way the dates were written 1895 - 1932? Then you quickly changed your thinking and "counted up" from 1895 to 1900 and added 32. What if the dates were 1923 - 1985? Did you unhesitatingly think 1985 - 1923, or not? Consider our country's birthday, 1776, and the War of 1812. Do you have a fleeting thought of taking 1812 away from from 1776 to figure the number of years in between?

I wonder what properties these number problems have that create for some of us momentary confusion in calculating time. Does it involve "place value" and the way we read dates (usually) as "eighteen ninety-five or nineteen thirty-two?" This might cause us to focus our minds on the number of tens rather than the number of hundreds, so we might think for a second of subtracting the smaller number of tens from the larger without due consideration of the hundreds. Or is there an inverse relationship between the number that represents the year and the age of what we are measuring which interferes with our thinking? For example, something made in 1895 is now 91 years old while something made in 1897 is less old. Does the idea that the smaller the number for the date of origin, the greater the age interfere with our thinking when we calculate that age? Or is it that when dates are quoted, almost invariably, the earlier date is given first? We say " from 1899 to 1912.." and then the first reaction is to write 1899 - 1912 as a subtraction problem.

For whatever reason, it seems that we usually start with the older or most distant past number and calculate "up" to the more recent date when computing time. Children often take the first number given and try to subtract the next number from it, right or not, especially with dates! Dr. Thomas O'Brien brought this to our attention with a "child - watching" project for a recent class. The project was to get children's reactions to the question, "If my

aunt has a birthday today, and she was born in 1898, how old is she?"

Two adults to whom I gave this problem immediately gave the correct answer by counting up to 1900 and adding 86. One eighth grader I tested quite spontaneously gave 88 as her answer, but could not tell me how she had thought it out. Two fourth graders and two other eighth graders approached the problem with pencil and paper in this way:

$$\begin{array}{r} 1898 \\ -1986 \\ \hline 912 \end{array}$$

and then changed their approach when they realized the answer was a bit unrealistic! One of the eighth graders reversed the positions of the dates, subtracted correctly, and obtained the right answer of 88. The other eighth grader wrote 99, 100, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, in vertical order, then evidently feeling assured that her reasoning was going to be correct, she gave 88 as her answer. The fourth grader who had given 912 as her first answer, worked at it again without paper and pencil and said "88." She then stated that she had "counted up" to 1900 and added 86. The other fourth grader knew that 912 could not possibly be correct, but was so baffled that she could not think of a different way that made sense to her so she gave up on the problem.

Another fourth grade girl very methodically wrote 1986 - 1898 in vertical form and computed the answer with "correct" regrouping techniques. She was "pretty sure" of her answer, but didn't think she could have worked it without pencil and paper.

A very bright fifth grade girl had a different method of obtaining an answer. She said, "I counted by tens from 1898 to 1988, then subtracted 2." I complimented her on doing it so quickly. She grinned and said, "I have a grandpa who is 89."

The one fact that characterized the thinking of all but one of these eight people is that each of them began with the smaller number and worked toward

the larger. Put another way, they started with the past and worked toward the present. This method resulted in total confusion for one child, and momentary confusion for three others. The latter three were confused only as long as it took them to reverse their thinking so as to come up with a reasonable answer. The one child who did not get a correct answer obviously could not reverse or reorganize her thinking. Does this fact have a relationship with Piaget's theory of the intellectual development of children? Children who are progressing through the concrete operational period of development may very well understand how to add and subtract whole numbers that are representations of physical objects, but numbers that represent something so elusive and abstract as time might be easily misunderstood. Even the eighth graders did not appear to be absolutely sure of their calculations involving time. Perhaps this also has something to do with conservation and reversibility, where reversibility involves keeping two distant points in time in mind in order to do a computation based on them. This involved, for some of the children, being able to think from the past date to the present, then in reverse from present to past.

There are implications for education in this little study, certainly. How can we expect children of ages nine through twelve to have an accurate perspective of the events of history when the concept of time is itself somewhat confusing?

MATH LIMERICKS

Diane Wooters

The winning team of Language Arts and Math can pack a powerful blow to boredom in the classroom. Limericks are enjoyed by all ages of children and are a fun way to work word problems. Limericks are written in anapestic meter. Lines one, two and five consist of three feet and rhyme; lines three and four have two feet and rhyme. These are sure to be a hit.

Trying to Reduce?
There was a young man with a pie,
Who cut it in eights. It's no lie.
He swallowed 2 slices,
And 4 more with spices.
What fraction is left of that pie?

Meter and Verse
There was an old woman who spun
A yard of gold thread for her son.
For her daughter, a meter
Of gold, sure to please her.
Which child was her favorite one?

Get the Point?
There once was a girl named Jezamal,
Whose waist was infinitesimal
Point 12 times point 2,
That answer will get you
The size of her waist and a decimal!

COUNTING IS APPROPRIATE FOR OLDER STUDENTS TOO

Linda Pauk

Counting is usually viewed as a skill to be practiced by primary students only. Most third and fourth grade teachers feel that their students know how to count and feel no need to use math time to review counting skills. I recently discovered there is a whole new area of counting to be experienced by these third and fourth grade students.

While reviewing counting by fives and tens with a fourth grade group, I decided to use a hundred chart and allow my students to find patterns with their counting. After allowing the students to mark the chart as below (Fig. a), I asked the students what would happen if they started with 3 and counted by tens. Most of the students looked at me and said that couldn't be done. We talked about it and used the board and addition to establish that 3 + 10 = 13 and 13 + 10 = 23. These numbers were than circled on the hundred chart and a pattern was established (Fig. b). As the next child counted by tens from 28, a student placed the numbers in a column on the board (Fig. c). The children then analyzed the column to see what was happening with the numbers. One child said that every number ended with an 8. Another student said that the tens place increased by one each time. After a period of discussion, the class devised a rule for counting by tens. The rule was to increase the tens place by one and leave the ones digit the same.

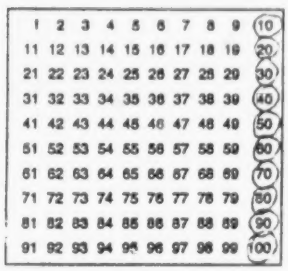


Fig. a

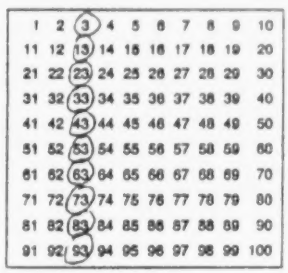


Fig. b

28
38
one greater — 48 — same number
58
68
78
88
98

Fig. c

The next step was counting by fives. The students first circled the typical 5, 10, 15, etc. on the hundred chart (Fig. d). We then decided to again start at 3 and count by fives. Addition problems on the board (3 + 5 = 8, 8 + 5 = 13, 13 + 5 = 28) gave us a pattern to circle on the hundred chart (Fig. e). As with the tens, a column of numbers representing counting by fives was written on the board and analyzed by the students (Fig. f). A rule of holding the tens place for two numbers and using repeating digits for the ones place was established by the students.

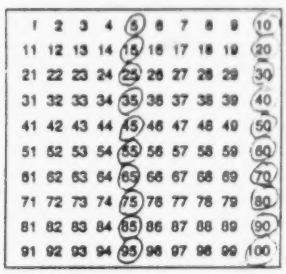


Fig. d

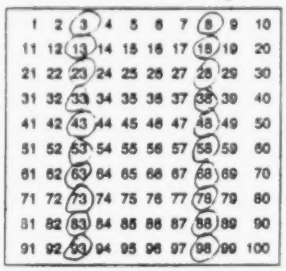


Fig. e

3
8
each 13 repeating
digit 18 sequence
used 23
twice 28
33
38
43
48

Fig. f

These counting activities were fun and exciting for the students. They were able to see patterns and used higher level thinking skills to develop rules for each counting task. Future work will include skip counting by threes and fours before we begin multiplication. The use of a hundred chart greatly enhances these activities. I would suggest a laminated chart so it can be quickly marked and erased. The most important factor to remember is to let the students mark the charts, find the patterns and devise workable rules. If allowed to complete these counting tasks, students will have a deeper understanding of our number system and how it works.

OPEN VS. TRADITIONAL SCHOOLS

Corolyn Elden

I would like to share my observations of my experiences in two diverse schools.

The first ten week period I was in a traditional school. The building was neat in appearance and decorum. It was somewhat institutional in design and atmosphere. When I walked down the halls the classrooms were quiet and the children were for the most part seated.

The only times the children had to interact were at recess and during lunch. At these times the children's behavior was unacceptable.

My second ten week period was in an open school. The building was covered with children's art. It was displayed at the child's eye level. The halls were also filled with projects which were in the works. One whole wall was devoted to a major art project.

On walking down the hall the classrooms were noisy. Children were interacting with each other. It was not chaotic as one would imagine. It was just busy and very active. The noise levels at recess and lunch were much lower than that of the traditional school.

On the whole I found the children to be friendlier, more cooperative and less inhibited at the open school.

The educational strategies are different in the open - vs - traditional school but why was the childrens behavior so different?

I did not address the daily curricula because in the traditional school I was with seventh and eighth graders and in the open school I was with first graders and feel it would be an unfair comparison. In just looking at the atmosphere, and physical structure of the two schools there were many differences. These differences had a direct relationship to students' behavior

and attitudes.

The closed, traditional school was somewhat rigid, children knew their place and stayed there. For young children to sit at a desk and do busy work with few breaks is a painful experience. Learning becomes drudgery. The very act of forcing children to sit for prolonged periods of time goes against their nature. Children are active, moving, busy people. They need to be stimulated and they need to move. Children who are forced to sit quietly are missing out on a big part of their needs. They become bored and frustrated and they equate learning as a difficult and painful process. When they do get the chance to move as at lunch and recess, they go crazy. It is as if they are going to explode and they probably feel as if they are.

Another thing I found was that when children are not allowed to interact with each other, to consult, to share ideas, they become competitive and somewhat hostile toward one another. There is no group mentality among the students. There is just each individual out for himself.

The open school was flexible with children. They were free to interact and move freely about the room.

The children were cooperative. They were concerned with the group and not just for themselves. They shared ideas and helped each other accomplish the days work. The children in this classroom were happy and friendly. Learning was fun and stimulating.

In the closed school, children were stifled, they are taught to conform to a set of rules which is diametrically opposed to their nature. The results lead to a painful school experience.

The open school is child centered. The children are allowed to be themselves and that leads to an enriching school experience.

"WHAT'S IN IT FOR ME?"

Amy Smolar

I'm not very good in school, grade wise, because I've had to stay in 5th grade twice. I'm a little bigger than some of the other kids but that's okay; I always get picked to be on a team real fast.

I don't think the teachers like me very much. They hardly ever call on me. They always talk about how important it is to read. I read stuff all the time; like car magazines, wrestling magazines, and Ninja books. I can pick out makes of cars and when I watch wrestling, I can tell you every kind of move they do before the announcer.

My teacher says those things aren't important and that I should concentrate on my school books. My spelling book has so many words I can never remember how to spell them or what they mean. I'm not sorry though, because the kids that know them real good were put in a contest and had to spell them in front of the whole school.

This year, I've been really interested in Ninja stuff and I've been trying to learn more about it. My dad says when I'm older, he might let me take karate lessons. That would be great because then if we ever get robbed, I could save my family. I wouldn't be a bully either. I saw some karate guys on television and they are in really good shape. They say it takes a lot of practice to be good. I tried to tell my gym teacher that we should do karate in class because it was good for

our bodies. She said she didn't want any trouble and that if I started anything, she would send me to the principal's office.

I'm not very good in geography either. They call it Economic Geography this year and we have to study about imports and exports of Taiwan. Maybe part of the reason is because I had to miss a couple of days when I helped my dad take some cows to the market and soy beans to the granary. My dad told me we were going and he said I had to figure out how much we were going to get for selling the stuff. I had to find the selling prices for the beef and soy beans from the newspapers and telephone calls to the stockyards. I made the phone calls about the prices all be myself and my mom said I did a great job. Boy, we made a bunch.

I had to give an oral book report once in class and the teacher said I spoke unclear and she couldn't understand what I was saying. She assigned the books and we had no choices.

I don't do very well in Math either. It just doesn't make sense sometimes. We had a problem that said: If a 57 foot telephone pole falls across a cement highway so that 17 1/6 feet extends from one side and 14 9/17 feet from the other, what would be the width of the highway? Why would someone measure a highway like that? And it didn't say if it fell straight across or not.

My dad says when I'm 16 years old, I can quit school and I can't wait. There is so much I want to learn!

WHAT HAPPENED?

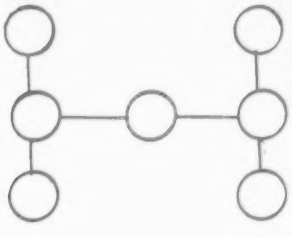
Martha Coderko

I've come a long way in regard to my philosophy and techniques in teaching math, and I hope to continue to grow and improve in the years to come. I enjoy teaching math this year. I've escaped the formal methods of teaching the basic skills through repeated use of commercially produced worksheets. Instead, I'm trying to provide experiences that are fun and at the same time nurturing students' abilities to think.

Each day I pose some kind of a problem that is a challenge; we call it **THINK**. For example, find 13 different ways to make $.25 using a quarter, dimes, nickels and pennies. Or, work with patterns to find the next three numbers in the sequence:

1, 5, 3, 10, 5, 15, 7, 20 __ __ __
1, 4, 9, 16, 25 __ __ __
1, 8, 27, 64, 125 __ __ __
J, F, M, A, M, J, J, A, S __ __ __

Or, use the digits 1 - 7 so that the sum of the numbers in each line equals 13.



I used dice games (games found elsewhere in **SEEDBED**) for teaching place values, comparing and ordering numbers, and rounding numbers. These activities were enthusiastically received by all members of the class. The students were in control; they were manipulating their own digits, making their own judgments, organizing their own facts. The students were having fun and learning at the same time. I had no discipline problems and everyone's

attention was on the lesson at hand.

Group work in teams has proved equally successful. Like most teachers, I dreaded the days we had story problem solving until I started group work. Each group is made up of 4 members with different mathematical aptitudes and abilities. I have kept the same groups since the beginning of the year and at this point they work very well together. The discussions that take place within each group involve more skills than I could ever introduce or challenge during a lecture-type discussion; verbal describing, making and checking of guesses and procedures, operational defining, creativity, classifying, brainstorming, estimating, eliminating, making judgments, confronting and resolving. I play the role of resource person, moving from group to group, offering advice whenever needed. As the year progresses, I find the team members relying on each other for help more than the resource person. Can I assume the students are beginning to take control and organize their own facts? Through this type of interaction, students realize there is more than one way to solve problems and a workable method is not invalid because it's different.

The calculator has proven to be very helpful in getting students to understand many basic concepts. For example, when introducing the concept of division, I use the calculator to show that the process is nothing more than repeated subtraction. The calculator challenges the students' understanding of decimal place value, for instance, change the 5 in 8.532 to a zero by using just one operation on the calculator. The calculators are also used by students as a means of checking their own calculations. Again the students are attentive, and in control as they use a manipulating tool that's going to organize their facts.

VARIATION ON LABINOWICZ

Diane Walther

Ed Labinowicz's new book, **LEARNING FROM CHILDREN; New Beginnings for Teaching Numerical Thinking: A Piagetian Approach.** (1985) contains a wealth of material on children's thinking and learning. The study borrowed, plus countless others, are included in his text. I would recommend it highly.

The situation in the interviews in the Labinowicz studies are similar to my mathematics class in that:

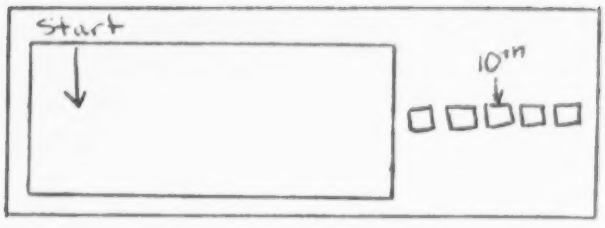
1. The children were second graders. (My children were also seven to nine years old.)
2. The task was given the third month of the school year. (The task was also given in November.)
3. The math instruction the previous year was traditional. (Our program was also traditional-Heath Math)

The situation was different in that I assume the 31 children in the Labinowicz study were of varied abilities. However, my class of 21 children appears to have much difficulty with scope, sequence, and pace in the traditional method of teaching mathematics, and they are currently working six months below grade level.

This task presented a method I could use to learn how my students think and to compare their attempts with a published study.

Quoting from Labinowicz: *"Task A examines how seven-year-old children take the order information given (tenth) and obtain both the unknown total of the collection and the quantity of hidden objects. It allows us to observe the extent to which these children are able to derive inclusion information from order information and to count-on and count-back."*

My variation: "A little while ago, I put some tiles on a board board. Some tiles are covered up with a shoe box lid. I counted the tiles starting from one end," (pointing to the left end of the lid). "One, two, da, tee, da, tee, da, up to this one," (pointing to the tenth tile). "It's the tenth one."



1. "What can you tell about the next one?"
   (Point to the eleventh tile.)
2. "What can you tell me about this one?"
   (Point to the ninth tile.)
3. "How many tiles are on the whole board?
   How did you decide?"
4. "How many tiles are hidden under the lid?
   How did you know?"
5. "Does the tenth one always come between the ninth and eleventh one?
   How come?"

All of my students were able to tell me that the tile after the tenth was the eleventh. Most were able to tell me about the tile before the tenth, although half of them called it twelve and then corrected themselves to nine.

The third question concerning counting-on provoked a variety of responses:

"How many tiles are on the whole board?"

Nine children were able to respond by counting-on, as Terrill did, pointing to each saying, "You stopped with ten, this is eleven and twelve. There's twelve."

Johanna and Kevin responded, "That's ten, and two more make twelve."

Some children could respond with my help. Pointing to the tenth tile, I said, "This is the tenth, (pointing to the eleventh tile) You told me this one was the eleventh, so..., (pointing to the twelfth) This one is..." Four of my students were still unable to supply the answer.

Jenny may not have understood the question. After my coaching (see preceding paragraph), Jenny responded, "five, cause five and five equals ten." Did she see five tiles and make up a problem equal to ten?

Aja guessed there were fourteen tiles. First, she counted aloud, starting with, "1, 2, ...9," pointing and tapping on the box lid. Then, touching the five tiles she could see, "10, 11, 12, 13, 14." She looked at the tenth tile again, looked at me puzzled. Going back to the box lid, she said, "They are spread out." Aja was relying only a spatial representation strategy. When her "10" count did not fit the tenth tile, she knew she was wrong, but she thought her mistake was in the "seeing" the arrangement under the box lid.

This thinking continued to question four: "How many tiles are covered up?"

Aja responded, "Six. More than five, cause there's more room."

Rose guessed 11, 15, and 17. I coached her. "Here's a clue, if this ten, and you said this was..." (I paused and Rose filled in "11".) I continued pointing to the twelfth tile, "so this is..." Rose said, "18." I took away the box lid and said to count all of them. Rose counted from one to twelve. Then I again pointed to the tenth, eleventh and twelfth tiles. I said, "This is ten, you count..." Rose counted, "11, 12." I summarized, "So, how many are there on the whole board?" Rose said, "18." I can think of two explanations for this response: either Rose has no conservation of these numbers, or she answered "18" because it was "her" number rather than the teacher's "12."

The fourth question demonstrates if the student's counting was reversible.

"How many tiles are covered up?"

Only six of my students were able to tell me they counted backwards, starting with the tenth tile, "10, 9, 8; there are 7 under the lid."

Ellen counted all the way back to one, saying, "Now I have to think how many numbers I counted."

I helped two children by asking them if they could count like a rocket blasting off. Pointing to the tenth tile and prompting, "This is ten;" then pointing to the ninth tile and waiting for the child to say "nine" and so on. Jenny counted silently (her fingers hidden under the table) all the way back to one. I asked her to tell me, "How many under the box lid?" In both instances, it took a very long time for the response, and both children seemed very unsure.

The last question explored the children's understanding of a simultaneous "one-more-than/one-less-than" relationship within the number sequence.

Joe said ten came between nine and eleven, "cause ten is in the middle. Like six and four, five is always in the middle."

Ellen said, "Nine, eleven, wouldn't be right; something would be missing."

David said, "Yes, it has to be in a certain order."

Johanna said, "It wouldn't sound right if the "ten" was out of line."

Labinowicz's task offered a rich opportunity for me to learn not only how each of my students think, but where each one needs to start the learning process.

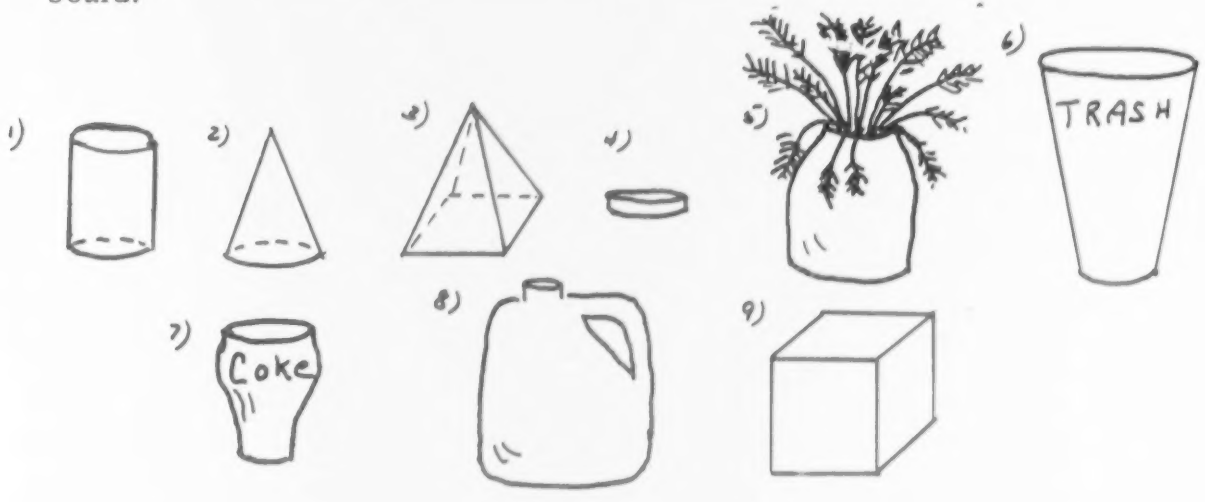
FORMULAS: TOOLS OR TRINKETS?

Barbara Madden

A formula is a powerful tool in the hands of people who understand it. In our math classes formulas are often dumped on students for them to plug in numbers and spit out answers. It's a mechanical process. No understanding is required.

My eighth grade math students had had plenty of formulas dumped on them. I was concerned that they were behaving as well oiled robots, cranking out answers to neat textbook problems. I decided to explore their understanding.

We had been studying the volume of prisms and cylinders. The students could solve the textbook problems using the rule V = B X h, where B = area of base and h = height of the shape. I drew the following shapes on the board.



I then asked them, shape by shape, if the volume rule was applicable. It was startling to me that there was unanimous agreement only in shapes 1, 4, and 9. These were the textbook shapes. In all other shapes class members

were divided.

We examined those on which we agreed. I asked why the rule could be used in shape 1, the cylinder. The students responded, "Because π X r X r X h equals volume....Because you take area of base times height....Because the book says so....Because the bases are the same." None of these were satisfactory answers.

We continued our examination. Students told me that the fourth shape was the same as the first. We looked at the last shape, the rectangular prism. Students explained that the rule applied "because it's not the same all over....The bases are the same....The book says so."

We then considered the two cylinders and the rectangular prism to find any common properties. The students found that in each case the shapes had congruent bases and the "sides were the same all over." We refined the "sides were the same all over" to the notion of "slices" congruent to the bases. We proceeded to examine the other shapes.

We looked at the milk jug. Students said the volume rule would not work because of the handle and the spout. I asked if the rule would apply if there were no handle or spout. They said the rule still would not apply due to the rounded corners.

We looked at the Coke glass. Students knew immediately that they could not form congruent horizontal slices, but wondered if vertical slices would work. They easily saw that because of the curved surface this would also be impossible. They also noted the bases were different.

The students easily saw that the formula would not be applicable for the remaining shapes. The cone had a single base, and the slices were circular, each one smaller than the last. The pyramid had one base with square slices, each smaller than the last. The waste basket and vase each had bases of different sizes, and different slices.

To summarize our results, I asked, "When can we apply the formula, V = B x h?" Some students quickly replied, "If there are two bases and they're the same size."

I asked, "Is that enough?"

Some said, "Yes!"

Jeff cautioned, "We'd better look at the sides."

But another added, "If the bases are the same the sides must be the same all over."

I said nothing, but drew this shape.

Students replied, "It wouldn't work because the slices are different."

I asked, "Then what do we need in order to apply this rule?"

Most said, "Same bases and same slices."

But Jeff said, "No, if the slices are all the same, the bases will also be the same. Just slices!"

When I first began teaching I was always frustrated when students were given the formula and still they couldn't solve the problem. The answer is really quite simple. Relationships are created, not packaged and delivered. The students must construct the rule themselves. The teacher must arrange the proper sequence of activities.

GREATER THAN OR LESS THAN

Debbie Johnson

As I was finishing my classroom observation I wanted to do something special for the children I had been observing. I asked my cooperating teacher if I could play a math game with them instead of their regular work time. She agreed and gave me their last 25 minutes. I had been observing them for several weeks, one day a week, and thought I could come up with something appropriate. The children had been working with worksheets on addition and subtraction up to five.

I'd gotten a hand out, "**Math Games For Children by Belleville 9 + 3**". In this handout was a list of games for primary children.

I decided to use this concept so I made sets of the materials listed.

Deck 1 - 3 x 5 cards with red dots of 0 - 5 (0 card has a red border)
Deck 2 - 3 x 5 cards with black dots of 0 - 5 (0 card has a black border)
Deck 3 - 3 x 5 cards with red numbers from 0 - 5
Deck 4 - 3 x 5 cards with black numbers from 0 - 5

GAMES

Greater Than or Less Than

Purpose:
Understanding of one more than and one less than.
Materials:
Decks 1, 2, 3, 4.
Number of Players:
Three or four
How to Play:

Shuffle cards and deal five cards to each player. Place the extra cards in center of table for players to draw from when unable to play. Players hold cards in their hands. First player plays a card face up in center of table. Next player places a card on top that is one less or one greater than the last one played. Continue in turn. If a card cannot be played the player draws from the center stack and the next player plays. First player to use all his cards wins.

Second winner is the person with the smallest number of cards when the game ends.

Comments:

As I was getting ready to give the children their instructions I became worried that the game would be too easy.

I showed the children the cards I had made and explained that the dots were equal to the number they showed, id: : : = 4, etc.

There were ten children in the class. I sat them at a round table and picked four to demonstrate the game, planning to break up in small groups after they knew how to play. After the cards were passed out, I asked one person to start . A three was showing. I asked him to look at his cards and place one higher or one lower on top of it. The whole group looked a little puzzled. I asked him "What is one more than three?" "I don't know, a five?" I went around the grou p and found they were all a little puzzled. I used my fingers and helped them figure out what number could go on each card. I was really surprised that they had no idea what I was talking about. After about five more minutes with that group, I made up two other groups with three people in each. I asked the children to try and help each other figure out what card they could put down. When I left the first group, they just started putting down the next card in their hand without looking at the card that was showing. The children were having a wonderful time, I became confused.

I thought, since they were answering their worksheets correctly, they would automatically know the difference between more than and less than. Boy was I wrong!

Then I read a chapter in **LEARNING FROM CHILDREN, "Arithmetic Notation and its Interpretation"** by Ed Labinowicz that children don't necessarily understand the symbols arbitrarily introduced to them. One of the assumptions I had been working on was that they understood equality sentences and their implied relations because they could successfully fill in the blanks. I asked one child what two subtract one was. He asked, "what does that mean?" As I thought about this episode I wondered if the novelty of a game, when they were used to worksheets, confused them, or was it unclear instructions which lead to their confusion. I settled upon the conclusion that they had little idea what one more or one less meant outside of the worksheets.

THE ENQUIRING CLASSROOM BY STEPHEN ROWLAND
Synopsis by Martha Coderko

After the Teachers' Center Session (Summer, 1986), I began to question my traditional approach to teaching in light of a more progressive method. Should I teach using the formal methods, emphasizing the basic skills through the use of commercially produced teaching materials or teach using the informal method, leaving room for creativity and inventiveness, developing techniques by applying them to real and novel situations. This debate led me to read **THE ENQUIRING CLASSROOM: AN APPROACH TO UNDERSTANDING CHILDREN'S LEARNING** by Stephen Rowland. This book gave me some new insights as to how children learn and how I, as a teacher, can best influence their learning.

The author observed and worked with thirty-three nine to eleven year olds, mostly white, English speaking, middle socio-economic group from Syston School District (six miles from Leicester, England). Rowland worked with another teacher so that he would have time to focus on individual groups and record his findings. The classroom had a certain degree of autonomy and the children exercised some control over their own activities. Outside the math and reading schedules used throughout the school, the teachers did not use commercially produced materials but preferred their own materials that met the interests and needs of the children.

I have made note of some of Stephen Rowland's detailed observations and interpretations mentioned throughout the book. All substantiate the fact that we as classroom teachers must share and carefully analyze our experiences with children if we are to have a good teaching and learning experience.

(1) If students feel compelled to accept the teachers interpretations and objectives, they will reject their own ideas and lose control over their own activities. They will always want to know what the teacher has in mind, and act accordingly.

(2) Teacher enthusiasm is good if it inspires independent learning, but BEWARE. Sometimes this enthusiasm can prevent the teacher from handing the control back to the students and thus make them too dependent upon the teacher. Teachers should guide students into new paths of knowledge but avoid holding on for too long.

(3) If the opportunity is offered, the students will make the most of it. The teacher will know when a student has gained control over his own work because he will start to ask: "I wonder what would happen if ---?" "How could this be so?"

(4) Teachers must be careful in introducing new techniques, so as not to assume the role as the one "who knows all the answers." This would only force further dependence of the student on the teacher.

(5) Rowland found that when the students had control over their own activities the benefit was not only an important factor in their learning, but provided him with an opportunity to understand how the children learned.

(6) During activities where the children were in control, the students showed a greater emotional commitment to their work. Their work had a greater significance to them than when they worked from textbooks, worksheets, and so on.

(7) Rowland found that when children controlled the situation, used their own language for their own purposes, were not required to interpret the ideas of a teacher, but worked with ideas that were meaningful and significant to them, they operated at the highest level of their reasoning ability.

(8) When students worked on highly significant and meaningful material, and invented their own rules, they had the ability to work at a high level of abstraction.

(9) The growth of mathematical understanding is not necessarily a linear

progression but mastered as a result of knowledge viewed in a new and different perspective. For example, one of the students wanted to weigh his caterpillar. Using a metric ruler as a balance and 1/2 gram cubes, the caterpillar was found to weigh more than 1/2 gram but less than 3/4 gram. Through this new perspective, the student learned to write and name fractions, learned various relationships involved in fractions, and learned how to add fractions. The student found the caterpillar weighed 9/16 gram. Because this student was working with meaningful context, and one in which he had the overall control, he was able to work at a higher level than what otherwise would have been expected.

(10) The teacher must constantly be interpreting the meaning of the students' activities to know when direct instruction is needed. But it is important to give the control back to the student at an early stage.

(11) The teacher is a resource person who the child can call upon when the need arises, and the ideas of each party are open to reinterpretation and change.

(12) Writing plays has an important role in the development of abstract thinking. Through their story writing, their thoughts are directed toward continuous critical appraisal. The child writer begins to gain control over his imagination, his environment, and his own intellect.

(13) Rowland suggests that to give the children today a better start than what we had, we should view their writing as an act of communication and not something to be graded and measured.

FINDING PATTERNS IN PAPERFOLDING

Linda Carroll

I passed out blank white sheets of paper - one to each second grade student. I told them to fold the paper 1 time (fold it double).

Teacher: How many parts do we have?

Children: Two

Teacher: Let's fold it again. How many do we have now?

Children: Four

Teacher: Let's make a chart to show what we're doing.

(On the board I drew this chart and went through it with them.)

number of folds	number of parts
1	2
2	4
3	?

Teacher: If we fold a third time, how many parts will we have? But, let's not really fold the paper again, let's just think about it. (Silence for about a minute)

Josh: 8 will go there. (in the circle)

Teacher: Why do you think so?

Josh: That side (pointing to the right side doubles every time.

Teacher: Then what happens on this side (pointing to left side)?

Two or Three children: It goes "1 - 2 - 3".

Teacher: Is "1 - 2 - 3" doubling?

Children: No!

Teacher: Let's try to finish the chart by thinking.

Teacher: If we could fold it 4 times, what would this be? If it doubled?

Children: 16.

Teacher: If we fold it 5 times? (pause)

Children: 32.

Teacher: Let's fold it 6 times! Some had pencils out figuring, but Josh was first as he jumped to his feet.

Josh: 64!

It was an exciting moment of learning to see eyes light up with understanding. Some, of course were utterly lost, but what of those who caught the vision? Josh, for instance, was almost gleeful in figuring out the answer. I could see him growing more confident each moment, more secure in his answers. What a worthy goal to help ignite that magic in each of our students!

MATH CLASS: OLD WAYS DON'T ADD UP

David G. Savage
Los Angeles Times Education Writer
Reprinted from **Los Angeles Times**, May 17, 1986

Here's the problem: "The 130 children at the Marie Curie School are going on a picnic. Each school bus can hold 50 passengers. How many buses will be needed to take all the students?"

Only one-third of the sixth-graders in California's schools gave the obviously correct answer - three. Instead, most said the answer was "2 and 3/5" or "2 with a remainder of 30."

"The first thing students do with a problem is get a pencil and start computing. That's what school emphasizes," said Tej Pandey, a testing official in the California Department of Education. "So it never occurs to them that you can't have a fraction of a bus."

For a growing number of mathematics teachers, this small problem and the children's reaction to it illustrates the odd mismatch between how math is taught and how it is used today.

SOLVING PROBLEMS

For adults, whether in business, universities or sitting around the dinner table at bill-paying time, the principal purpose of math is to solve problems. Simple ones arE done in the head, and detailed computations are performed on a calculator. For students in the middle and upper grades, however, math classes consist mostly of doing computations by paper and pencil.

"We spend 95% of our time having them practice something that won't be used outside of school," said Clyde Corcoran, a Whittier High School math teacher who chaired a statewide panel seeking to redirect the math curriculum.

Typically, students do not grapple with more challenging math problems that require thinking not just computing. At the same time, they often miss out on learning such practical skills as how to use statistics or how to mentally estimate an answer.

"We do spend a lot of time working on long division, which is ridiculous," said Marian Pasternack, a math teacher at the Sherman Oaks

Center for Enriched Studies, a Los Angeles magnet school.

PUSHING FOR CHANGE

These math teachers and others like them are pushing hard to shift how math is taught -- away from paper-and-pencil computations and toward solving more complex problems, possibly aided by calculators or computers. And this summer, they plan to make their voices heard as the California State Board of Education begins its review of the new math textbooks to be used in the state's public schools over the next seven years.

Theirs is neither the abstract "new math" approach of the 1960s nor a "back-to-basics" push from the 1970s. Rather, they say they want to bring mathematics teaching into line with a world of computers and calculators.

The effort is being watched closely by many mathematicians who believe that the math curriculum is archaic. Because California is one of the few states to select textbooks on a statewide basis - and is the largest buyer of them - the outcome of this debate is expected to influence mathematics teaching across the nation.

"The traditional curriculum in the middle grades was designed for a nation of small shopkeepers," said Henry O. Pollak, a mathematician for Bell Communications Research who headed a National Science Foundation panel on school mathematics. Yet, for shopkeepers as well as almost everyone else, the nearly universal use of calculators and computers has changed what they do.

"The calculator has made some subjects more important and some less so," Pollak said in an interview. For example, being able to estimate has become more important. "The best defense against a silly mistake is to know how the answer should look before you begin," he said. A student should realize, for example, that multiplying 19 X 22 is close to 20 X 20 so the answer should be close to 400.

"It is also important that citizens can look at data, analyze it and draw conclusions from it. We don't teach much of that, " he said. "Instead, we spend an enormous amount of time doing paper-and-pencil arithmetic."

Pollak's panel, in a 1983 report to the National Science Foundation on "What Is Fundamental" in mathematics, urged schools to introduce calculators into the math classrooms "at the earliest grade practicable." Moreover, the panel said, "substantially more emphasis should be placed on [teaching] mental

arithmetic, estimation and approximation, and that substantially less be placed on paper-and-pencil execution of arithmetic operations."

The just-published California Mathematics Framework, the guide for choosing the state's new textbooks, adopts the same basic view.

"The California framework is terrific. If they put this into effect, California will lead the nation in changing how mathematics is taught," Pollak said.

Since hand-held calculators first came into broad use in the early 1970s, many teachers and parents have viewed them as a threat. If a tiny computing machine is available as a crutch, how will students learn basic arithmetic?

But for more than a decade, university researchers have studied pupils who have used calculators and have found no dire consequences - and some significant benefits.

"The clear indication of the research is that the calculators have a positive effect on achievement," said Marilyn Suydam, an Ohio State University professor of mathematics education who headed a federally funded center on calculators and learning.

Of 175 studies comparing classrooms where calculators were used with those where they weren't, "less than 2% showed a somewhat negative effect on achievement," she said. In about half the studies, the students who had used calculators in class scored higher on math tests. (Neither group of children was allowed to use calculators on the tests.) In the rest of the studies, there was no significant difference in the rate of learning between the two groups, she said.

"It is important to remember we're not talking about an 'either-or' situation. The children still must learn the basic facts in their head, and they continue to do most two-digit computations on paper," Suydam said.

The studies also turned up a less surprising finding: Relieved of the tedium of doing long computations on paper, students in the calculator classes reported that they liked math more.

UCLA mathematics Professor James Caballero noted that most new inventions have aroused a similar fear.

"It is often said that the invention of papyrus caused fear that memories would dry up because people could write everything down, or that, during the Reformation, books would allow people to read others' thoughts and

therefore, not learn to think for themselves," he said.

There are some math teachers who believe just that.

The 40,000-member National Council of Teachers of Mathematics has strongly endorsed the use of calculators in the classroom, but in April, its annual meeting in Washington was the scene of one of the odder demonstrations in the nation's capital. Decked out in a navy blazer and a straw boater, John Saxon, a conservative publisher of his own math texts, led 20 teachers carrying protest signs outside the convention hotel. They read: "The Button's Nothin' 'Till the Brain's Trained" and "Beware: Premature Calculator Usage May be Harmful to Your Children's Education."

QUICK TO AGREE

But the advocates of using calculators in schools were quick to agree with Saxon's point that students need to learn basic arithmetic on their own.

"The calculator is a tool for someone who already knows basic arithmetic," Suydam said.

Although calculators have been slow to work their way into the classroom, math professors say the main obstacle is no longer teachers. Instead, they blame the textbooks and tests, both of which continue to stress the traditional paper-and-pencil exercises.

"For most teachers, the textbook determines what they do in class," UCLA's Caballero said. "We're hoping this new framework [the state guidelines] will cause someone to generate some new and better texts. A lot of us are optimistic that something really significant will happen this time."

Last year, the state board rejected the pending series of science textbooks, insisting that the works be revised to include more material on controversial topics such as evolution and human reproduction.

EXPECTS SIMILAR FIGHT

State Superintendent of Public Instruction Bill Honig said that he expects a similar fight on mathematics books.

"We want math books that are more challenging - just better books - which is exactly what we fought for in science," Honig said. "We have some big holes in our [math] curriculum now. We need to teach computation, but we also need more work on problem-solving, geometry, measurement, and

statistics."

A team of mathematics teachers and professors will review the new series of math texts over the summer and make their recommendations to the state board in the fall.

"We've set forth the guidelines on what we want, and we're going to stick to them," Honig said.

To go along with a change in textbooks, the state Department of Education is working on a change in its tests. Teachers have complained that standardized exams in math focus almost exclusively on computation.

REASONABLE ANSWER

As a result, the mind-set for students is "get a pencil and start computing, regardless of what the problem says. We want them to think first about what is a reasonable answer," said Pandey, the state testing official who is working on a redesign of the state's 8th-and 12th-grade exams.

State officials believe that they can get across that message with a simple stroke. Beginning in December, 1987, 12th-graders will be permitted to use hand-held calculators on the math portion of California Assessment Program tests.

The test designers are busy putting together new questions, some of which are more complex problems while others will challenge the student's ability to mentally calculate the best answer.

The mathematicians and state officials hope that new textbooks and redesigned tests will convince even the reluctant teachers that it is time to change the way math is taught.

"The kids basically learn what we teach them, and what we've been concentrating on is routine drills in computation," said Corcoran, the veteran Whittier teacher. "But we've totally ignored what's been going on outside the schools. Our curriculum is outdated, and we're turning out students who are not prepared for a world of computers and calculators."

"In education, it seems to take decades to change anything," he concluded. "I just hope we're going to make it this time."

PIAGET PRIMER: BOOK REVIEW

Jane Stump

THE PIAGET PRIMER, Ed Labinowicz. Addison Wesley Publishing Co., Menlo Park, CA., 1980.

In an effort to bridge the dichotomy between Piaget's theories of learning and American teaching methods, Labinowicz's text uses many eye and mind catching techniques. With large, boldface print, Piaget's thoughts are emphasized clearly and with detailed, realistic drawings interspersed throughout to illustrate teaching techniques and learning patterns. **THE PIAGET PRIMER** strives in its effort to "convince" the American teacher of the logic and soundness of Piaget's learning theories.

Before plunging into the theories the reader is presented with an exercise in water displacement, involving children, to show the differences between adult and child thinking. "Are these children thinking?" poses Labinowicz. "Are they learning? If so, what evidence is there that the children are learning? Couldn't the teacher be teaching?" By answering his questions with typical "teacher" answers, and then posing affective questions to the reader, he leads into the personal background of Jean Piaget, and how his theories were developed.

In an era which considered children to be miniature adults, from the way they were dressed to the way they were considered (by adults) to think, Jean Piaget recognized children as completely different from adults. Instead of asking children questions and tabulating only the correct answers to formulate a standardized test of reasoning, Piaget listened to children and found patterns in their "wrong" answers. This discovery led to a lifetime study for him, as he set a precedent of accepting children as they were, without imposing adult expectations or explanations. To quote Labinowicz, "Although he is interested in what children know, Piaget's chief concern is how children come to know ... Children of like age groups responded in ways that were remarkably similar to each other and remarkably different from adult responses and expectations." (p. 21)

One idea that repeatedly emerges emphasizes the child's mind is not a camera, and that children have different ways of determining, figuring, and organizing ideas, or of recalling something from their memories. These ways of learning change as the child matures in body, physical experiences, and social interaction - but none of the factors in themselves explain intellectual development, according to Piaget, who adds a fourth critical, coordinating element: equilibration, or self-regulation.

Equilibration occurs as a child's mind interacts with reality, fitting experiences into what he already knows, and at the same time, adjusting what he knows in response to his experiences. "The child is the mainspring to his own development," states Labinowicz ..."The child's activity, not only uncovers new problems, and this initiates disequilibrium, it also constructs a solution and attains a higher level of equilibrium." (p. 46) During disequilibrium, when a child is reorganizing and reconstructing patterns of thought, a paradox occurs. What a child thinks and what he does doesn't always agree; yet, states Labinowicz, "Often children who experience the most confusion ... arrive at the highest level of understanding." (p. 55)

It is the teacher's duty not to assume to know what her children are learning, but to evaluate their understanding of concepts by challenging them with different (similar) situations until she can observe not only mental engagement with the concept, but also observe the children gaining mental satisfaction, because learning involves a great deal of time. What a child assimilates today, he might not accommodate for several more days or weeks. While the chart in the Teacher's Edition of the math book might say "they're ready now for the multiplication tables of 2 - 5," teachers would be reminded by Piaget that teaching doesn't always mean telling.

It is our duty as teachers to help our students to stretch their thinking, for teaching is preparing a child for life-long learning.

A DISCOVERY

Nancy Sweet

Eight year old Josh Muskopf, in my 3rd grade math class, discovered something in the times table.

9 X 1 = 9 Put the factors 9 & 1 together to get 19. Subtract the factor 1 & you get the next answer, 18.

9 X 2 = 18

9 X 3 = 27 29 - 2 = 27

9 X 4 = 36 39 - 3 = 36

9 X 5 = 45 49 - 4 = 45

9 X 6 = 54 59 - 5 = 54

9 X 7 = 63 69 - 6 = 63

9 X 8 = 72 79 - 7 = 72

9 X 9 = 81 89 - 8 = 81

Neat, huh?

COMPUTERS -- COMPLEX THINKERS SOFTWARE

Shirley Gasparich

Observations of "Safari Search" Activities

Computers are definitely an "in item" in schools today; most schools boast of having at least several--though few teachers use them much for meaningful learning activities. The software usually consists of games children play which promote "trigger-finger" action rather than complex, critical thinking. The primary goal is that the software be user friendly and acquaint the students with the computer keyboard.

On the contrary, I have had the wonderful privilege and experience of exposing a group of students -- 3rd, 4th, and 8th graders -- to a software program designed by Dr. Thomas O'Brien.

Called **Safari Search,** the software activities help students develop complex problem solving strategies, logical thinking, and inference skills. The diskette contains twelve search activities on a 5 X 5 grid, from which students collect their own data, construct hypotheses, and develop complex judgments. Most importantly, it is challenging and fun for a wide range of age and ability levels from grade 1 or 2 to adults. In addition, the activities foster growth and development in many other areas as well. Other aspects of these software activities I found particularly conducive to fostering students' learning are as follows:

- They encourage students' thoughtful, inferential thinking skills rather than rewarding their impulsive, key-punching behavior.
- They are easily played by groups of students, thereby promoting group cooperation and problem solving.
- Group work facilitates student's growth in social skills, as well as enables them to benefit from others' thinking strategies and viewpoints.
- They promote the students' language development and communication.
- They are successfully played by students of mixed ability levels.

The first six activities are within the reach of first and second graders, while the last six activities present situations even adults find challenging.

First I worked with a group of third graders. They played 3 games: Search Out the Seal, Locate the Loon, and Find the Flamingo. The rules and strategies vary from one activity to another. My observations are as follows:

Find the Flamingo

Job: To Find Fred the Flamingo.
Select a box. It will indicate the following:
WARM - if the box selected is touching Fred's box., cornerwise.
HOT - if the box selected is touching Fred's box, sidewise.
COLD - if the box selected does not touch Fred's box at all.
Any number of clues may be selected.
Press (R) for a review of collected clue data at any time.

HOT		HOT	COLD	
WARM	HOT			

Game 1: The students used 6 clues to find the flamingo.
Game 2: They found Fred in 4 clues.
Game 3: They found Fred in 2 clues.

Since the screen shows the information contained in 1 box only, it encourages the students to keep several issues in mind simultaneously -- an important as pect of complex thinking.

By the end of the third game, my original four students had swelled to 12 or 15 others who had left their computers to enter the stimulating debate about where the next clue should go. There was a maze of fingers pointing to various squares on the computer screen and excited children were voicing their opinions, trying to convince others. The language, reasoning, and debating skills were exciting to listen to and observe. I was especially moved by the extremely good strategies voiced by one little boy who had been retained. I couldn't help commenting to the others about what good ideas he had. He was obviously proud of himself, too. Another very shy, reserved girl occasionally had a good suggestion, as well. I felt compelled to point out to the students when I heard some especially good reasoning expressed.

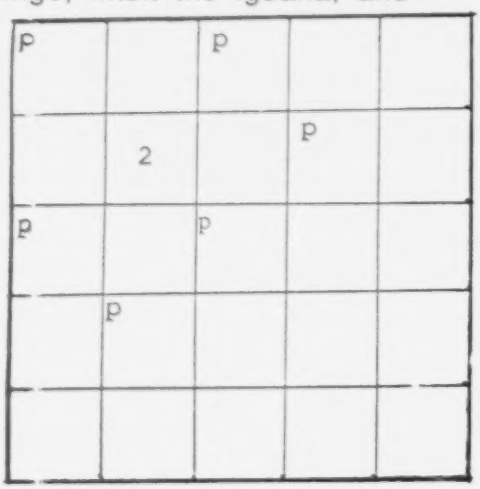
The happy glow on the children's faces as they left the computer lab said it all. They had had a very rewarding, stimulating, educational experience. It would be an understatement to say that the software activities were of a high interest level. The kids loved them!

I was reminded of Caleb Gattegno's view that students, teachers, and learning situations need to make use of powers of the mind other than retention. Transmission of stored knowledge and rote memorization dominate today's educational curricula. Why do all this memorizing of "***STUFF***"? Students need to be learning how to **THINK** in complex and creative ways. Knowing how to think and solve problems is what is going to carry them into the 21st century equipped to handle all the unknown problems and adventures that await them.

Next, I worked with five fourth graders. They were relatively sophisticated computer users; some have home computers and all have two or three years experience using them in the school computer lab. They played four of the activities: Search Out the Seal, Find the Flamingo, Intuit the Iguana, and Discover the Dragon.

Search Out the Seal

Job: To find Sammy the Seal who is hiding in one of the boxes.
Select a box. The distance will be given from the box selected to Sammy's box. Only up/down and right/left distances will be given (No diagonal distances) Any Number of clues may be taken. Press (R) for review of collected data.



Usually, the students could readily see 2 or 3 possibilities. However, after studying the grid for a while, someone would usually surprise the others with an additional possibility. Once shown, it seemed all too obvious, but had, nevertheless, been overlooked. The unstated objectives of this activity were to see relationships, develop hypotheses, and select clues that would eliminate and/or give information about as many boxes as possible, rather than make isolated guesses. Again, the students were encouraged to keep many issues in

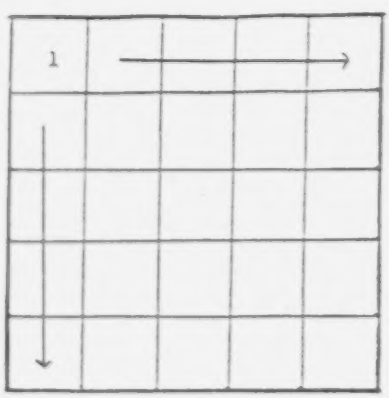
mind simultaneously. They were challenged to see several possibilities and alternatives which was facilitated by having several students working together. This enabled them to benefit from sharing and listening to other's perspectives and opinions, expanding their own personal viewpoints and strategies.

After selecting the middle box for the first clue on several occasions, one girl suggested that, "For this game the middle box is very confusing. If a small number is given in the box, there are too many directions which are possible. The middle box strategy had been helpful in an earlier activity. The children also learned that if not considered carefully, a second or third clue eliminated only 1 box, or, in some instances, provided little additional new information. Without any instruction they were constructing strategies that worked for them. They found Sammy using 3 clues.

I especially liked the way these activities encouraged thoughtful, deliberate thinking about their possible choices, rather than swift, impulsive, random guessing -- which brings up another issue. One student felt so confident about her own opinions that while the others were debating the next move, she simply moved the X and selected another clue herself. Furthermore, while the rest of the children were reading the game rules out loud, she advanced to the next page since she had already finished reading them silently. Of course, the others hadn' t finished yet. Needless to say, these actions brought on some pretty intense protests from her "computer-mates." She soon learned they would not tolerate her impulsiveness, inconsideration and egocentrism.

DELORES THE DRAGON

Job: To find Delores the Dragon who is hiding in a box.
Select boxes to obtain clues. The computer will scan the row (left-right and column (up-down) in which your box is located. You will be told if Delores is seen in the Row/Column scan. Select as many boxes/clues as you wish. You have only 1 chance to discover the Dragon. Press ESC Key when you know where Delores is hiding.



(If the dragon is in the top horizontal row, all the top boxes will say 1, if clues are asked. The students must still determine which box contains the dragon by asking clues and using inferential/deductive reasoning.)

This was a very challenging activity for these 4th graders. (Remember: the impulsive key-puncher was still in the group!)

11 Clues:
"I give up!"
"Wait! Don't press escape! Not everybody is ready to give up!"
"We already tried that!"
"Review! Review!"
"It's got to be in this place, because that's where it crosses!"
"No! Wait! - - - Because ..."
You're not supposed to give up!"
"It has to be this one because the whole row says "1".
"Review! Review!"
"It has to be here!"
"Does everybody agree?"
"Wait! Just because most of you think it's here, doesn't mean it's right!"

We stopped for a moment to discuss the merits of majority rule versus wise decision making. The students decided that without good reasons to justify it --- just because several people wanted to do something didn't necessarily make it the wisest choice.

Suddenly --- with a total of 22 clues, after some intense debating, and with some genuine social issues contemplated and discussed, they were rewarded by finding Delores.

INTUIT THE IGUANA

Job: Find the Iguana hiding in the box.
Select a box; the computer will tell you Y/N if the Iguana is hiding in that particular box. Select as many clues as necessary. (This is the first game on the diskette. No clues given about relationships between or among boxes. One simply guesses. Remember: These students had just engaged in some intense debating and rather complex problem solving.)

	NO			
		NO		
	NO			
		NO		

Dialogue:
"This is aggravating, isn't it?"
"All you do here is guess!"
"This is boring!"
"It's more fun when you get more clues and can think ... figure out where it's hiding!"
(I had purposely skipped this game, thinking it not challenging enough for them. However, they were intrigued with the title and insisted on trying it. I'm very glad they did. We both learned from the experience.)

THINKING IS MORE FUN THAN GUESSING!
MAJORITY RULE VERSUS WISE DECISION MAKING!
GRAPPLING WITH RELATIONSHIP ISSUES!
DEALING WITH SOCIAL DYNAMICS!
COLLECTING DATA , MAKING INFERENCES, DEVELOPING AND TESTING HYPOTHESES .....

These issues seem like pretty heavy duty stuff for 9 year olds who had just planned on playing some computer games.

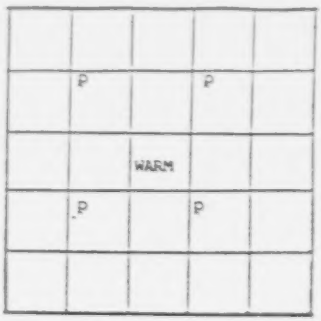
In **WHAT WE OWE CHILDREN**, Caleb Gattegno presents a thought provoking issue: ***"Time is the Stuff of Life"*** --- Do classroom learning situations give students and teachers break even returns on their investments?

In my opinion, these students and I were richly rewarded for our 45 minutes together.

In marched four eighth grade boys. They, too, had had computer experience. They wanted to see the really tough stuff! However, in their eagerness to get on with the games, they misread the rules.

FIND THE FLAMINGO

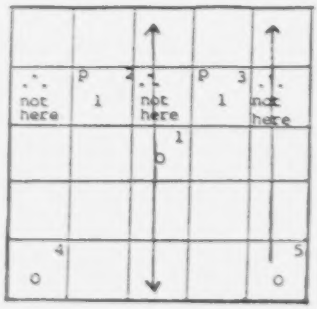
Game 1 Dialogue:
"Put an X in the Middle"
"It will eliminate the whole row."
"Wait! I don't think we understand the directions!"
They immediately discarded their first game plan and continued playing, constructing and inferring rules as they obtained clues. They were able to handle the uncertainty of not knowing the rules quite easily.



Game 2 Dialogue
"So let's go with the middle again!"
"It covers the most area. "
"Warm!" ... These are the possibilities. (pointed to the 4 corner squares adjacent to the "Warm" square).
"Let's try here. Got it!" (used 2 clues)

Game 3
"Pick the middle again. Hot!"
"It's one of these 4. (pointed to the possibilities.)
"If this is cold, ... it's cold!"
"Then, you know it's not these two. (Pointed to boxes above and below "Hot" box.) Otherwise, it would say "Warm".
"It's got to be here! Got it!"

DISCOVER THE DRAGON:



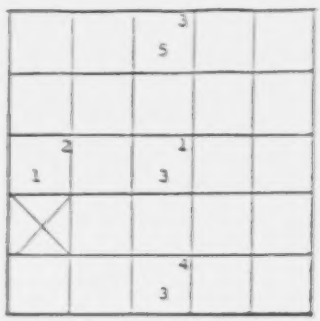
"He can only be in one of these two boxes;"
"We've eliminated these columns because they say "no".
"Let's try the top."
"Wait! Why there? Oh, yeah! He can be

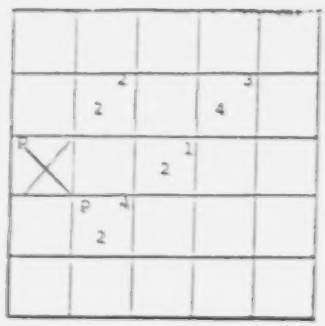
here or here!"
"He's in this line! Wait! Is there only 1 dragon?" (unlike younger students, these 13 year olds were able, willing, and wanted to consider every possibility.)
"Got it!"

SEARCH OUT THE SEAL

They now read all directions together, aloud, having experienced their previous rule misunderstanding. (They corrected their own mistakes on their own.)

Game 1
"Go to the middle; it covers the most!" (At first they incorrecllv inferred the clue number given in the box included that box as number 1.
They considered the erroneous possibility of their inference after clue #2. Af ter clue #3, they were positive of their error. Then they inferred that the distances were not just straight lines, but could be L-shaped as well.
Next inference:
"It gives you the closest path!"
"This is it. Got it!" (used 4 clues)





Game 2 Dialogue after 3 clues:
"It could be these two or these two."
"We want to devise a way to check ..."
"No!"
"Because it goes in the shortest path!"
"If we go here, then ..."
"Check here!"
"Ok, it can only be here!"

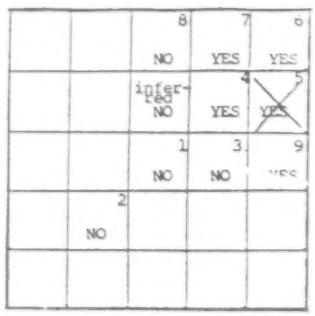
They were very efficient and confident in their problem solving -- collecting data, keeping several issues in mind simultaneously, developing

hypotheses, and making inferences through constructed negation. These 13 year olds were obviously operating on a much different reasoning, social, and priority level from the 8 and 9 year olds. They had already worked through the cooperation issue which had dominated much of the 4th graders activities. Each group engaged in similar complex thinking activities, but they did so on very different levels, (certainly an attribute of the software's usefulness and flexibility.) I asked the fellows to comment on the activities they had engaged in thus far. Here are their opinions:

"It's a challenge."
"It makes you think."
"It would be good for little kids; it would make them think. It makes us think!" :

LOCATE THE LOON

Job: To find loon hiding in a box.
Select a box. Yes = the loon is hiding in or is touching the selected clue box. Select as many clues as necessary. You have only 1 chance to find the Loon. Press ESC Key when you have found the Loon. Press "R" Key for collected data review.



Dialogue after 5 clues
"This is going to be Yes!: (6th clue top right)
"And the one next to it is yes!" (7th clue)
"Let's review!"
"You guys, if this is no ... we try this!" if this yes ... then, this is the box!" (5th clue)
"This is it!" Everything matches up!"
(They found the Loon using 10 clues.)
Throughout the activities, these fellows made inferences and hypotheses, but quickly generated new alternatives when their hypotheses didn't work out. Certainly, the ease with which they moved from one thought pattern to another exemplified the great flexibility of their thinking.

CATCH THE KITTENS

Job: To find the 2 kittens hiding in the boxes.
Select a box. The computer will give you the distance from your box to one of the kittens. It won't be the same kitten every time. You won't be told which kitten it is. You have only 1 chance to catch both of the kittens.

In game 1, they found one of the kittens, but incorrectly inferred where the second one was located.

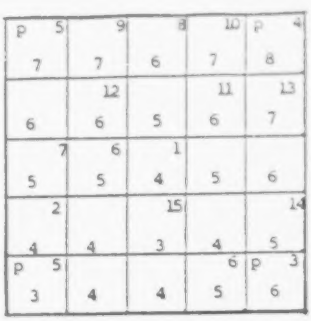
In game 2, quickly learning from their mistakes, they found both kittens using only 3 clues.

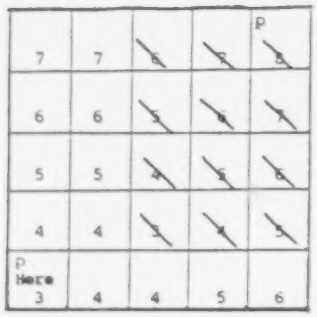
Since they were eager to be really challenged, they moved on to the last of the 12 activities, skipping several intermediate level games.

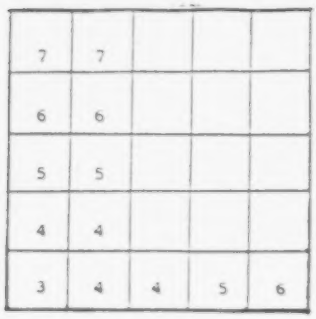
LOCATE THE LLAMAS:

Job: 2 llamas are hiding, find them.
The computer will give the distance from the box to the nearest llama and from it to the second llama. Only (up-down) and (left-right) distances are given. Select as many clues as necessary. You have only 1 chance to find both the llamas. (They uncovered every box, but still weren't sure where both llamas were hiding.)

Game 1
"There could be a llama here. (top right box) It could then be 8 to the other llama."
"One is in a corner because 4 in any direction is a corner."
"Go here!" (bottom right)
"How could it be here? This doesn't match up!"
"Everything else matches up"
"I don't think they would have 9 clues go to one square."
"We know one's here!" (pointed to bottom left)







This activity involves some very complex thinking, and was quite a challenge for these 13 year olds, who had easily moved through and quickly mastered several of the earlier activities. They felt they had found 1 of the llamas, but the bell rang before they could find the other one. One student drew the 5 X 5 grid on a piece of paper and quickly eliminated all the number clues that referred to the box in the bottom left corner. (The location of 1 llama, so they deduced). He reasoned that the other clues had to refer to the second llama.

If intellectual maturity is the acceptance of lack of closure, then these fellows experienced it first hand. However, they were most reluctant to go to their next class. Throughout the 45 minute period, they had been stimulated, challenged, and actively involved in some very complex, critical, and creative thinking.

To be sure, they were involved in a very intense, logical mathematical experience --- obtaining knowledge not through simple observations, but by personally constructing it through negations.

What do we owe our students? I suggest they deserve opportunities such as these pupils experienced:

- Solving complex problems
- Working cooperatively in groups
- Exchanging view points
- Developing and testing hypotheses
- Making inferences
- Constructing negations
- Engaging their minds in complex, creative, problem solving activities.

Indeed, as Caleb Gattegno states, ***"Time is the Stuff of Life"***. Our students deserve the best return on their investment we can possibly offer.

A SHAPE PUZZLE

Sue Nash

I substituted in a Learning Disabilities Resource Room recently and had an opportunity to work with two and three students at a time. When they finished their assigned work, I then asked them to try to solve this attached puzzle.

Because of lack of time and to lower the level of frustration for these students, I had previously cut out shapes and paper clipped them together.

I showed each student the original page and explained to them that they were to use a sheet of the green paper and arrange the shapes held by paper clips into a new design or object. I requested that they tell me what they had created or to explain what it could do.

They were told to make many of them, if they had time. Only Jenny made 3 different pictures. Even though different - she retained some sameness: 1) sun in all pictures, 2) bricks in all pictures, 3) trees and bushes in two pictures. The differences were important though, because she created three completely different objects: robot, dog, house. This girl came to the L.D. Resource Room to receive help with her listening and visual acuity; however, in this particular activity she comprehended the instructions and followed through. I found it interesting that she added two suns to two of the pictures. Jenny, a 2nd grader, worked 30 minutes.

Vincent did not want to try this activity. He is extremely hyperactive and has a difficult time sitting in his chair long enough to think about what he does. He came to the Resource Room to help with visual and listening acuity. It took him 30 minutes to accomplish this one picture and at that, he didn't accomplish the task assigned and refused to think any further than the shapes he used. I encouraged him to use the rest of his shapes and he out-right refused - finally throwing them away and then exclaiming to me that he was finished. He is a 2nd grader.

Jeremiah is also a 2nd grader. He completed only one picture, but stayed occupied 30 minutes creating it. I find the off-balanced shape of the robot interesting; but I see balance in the chaos. In my opinion, this is very creative for an L.D. child who has difficulty with listening and visual acuity.

He also has a behavior disorder.

Jim came at a different time than the other 2nd graders I worked with. He was in the Resource Room at the same time as a 5th grade boy, Thatcher. They both displayed behavior problems and seemed to have had a distinct dislike for one another.

Jim told me immediately upon meeting that he had to go to court the following day because his mother and dad were fighting over custody of he and his brother. He seemed like a little old man when he talked about his future and the court hearing. His mother had previously kidnapped the children and when the police caught up with her they were returned to his dad where they were currently staying.

He was very upset but cooperated in doing his assigned work and this attached activity. I noticed the detached state of this boy - I'm sure it does not take a psychiatrist to figure out the connection. He made almost the identical object as the original page but connected all parts except the head, hat, and hands. He straightened the legs out so that the object was standing squarely on the plane.

I asked him what he was making after he had pasted a few shaped and he said in an aggressive way that it was going to be the CLOWN, Thatcher - which, of course, got a rise out of Thatcher. I asked Jim to make a nice story (Thatcher had told me he goes to drama classes in Alton and had been to the SIUE drama festival). I encouraged Jim to see Thatcher as an actor clown and make an acceptable story, which I copied down as he told it to me.

Thatcher is a delightful, rather hyperactive boy in the 5th grade. He enjoys dramatics and his parents drive a long way each week for him to participate in drama workshop classes. He came to the Resource Room to receive help in English and Reading. He had some speech problems and commented to me about them.

He worked cooperatively, and quickly completed picture #1 which he entitled Mickey Mouse in the Forest. His picture depicts himself as front and center stage. Balance seemed important to him.

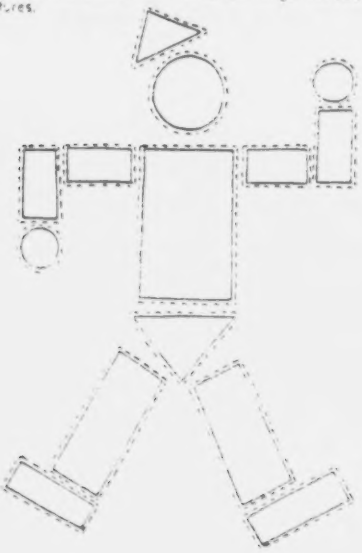
On picture #2 he wanted to make a picture of Jim (above) who had earlier made fun of him. I encouraged him to forget about Jim and he created the giraffe. He refused to use extra puzzle pieces.

I CONCLUDE THE FOLLOWING:

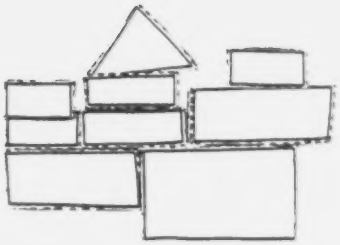
1. These children had Learning Disabilities that interfered with certain areas of their learning; and these disabilities in all cases seem to affect their ability and patience in problem solving strategies.
   a. Jenny did well for her ability, but did not do inherently different things with her shapes. Her pictures are balanced much the same as the original.
   b. Thatcher did O.K. on first effort but did not want to think through another effort - his patience was gone and he denied the reality of the exercise.
   c. Jim did well for his ability and particular set of problems - I do believe he had the best problem solving ability.
   d. Jeremiah was more creative than the others but perhaps it was by accident. I wish he had tried another.
   e. Vincent seemed to have more pronounced problems in listening and visual acuity and also behavior problems. I feel these factors hindered him from entering into the activity with any degree of excitement. He had not heard any of my conversation with Thatcher, so I don't know what his experience with the stage was - but it was obviously on his mind. I though it was a rocket ship but he quickly informed me that he had "written" a stage on his paper.
2. I thought there would be more abstractness in using these shapes, however, each picture was, basically, realistic in types of objects created. I had expected experimentation with creating new machines, designs, etc.
3. In doing this activity again with regular classroom children, I will do it one day and then return to the same activity a week later and ask children to predict how many different types of pictures we could make from shapes. I will then challenge them to try their prediction by creating that many.

NAME _____

Change this person into something new by moving the shapes. Cut them out and use them to make three new things. Tell about their adventures.

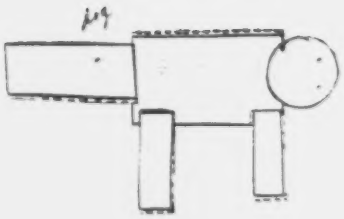


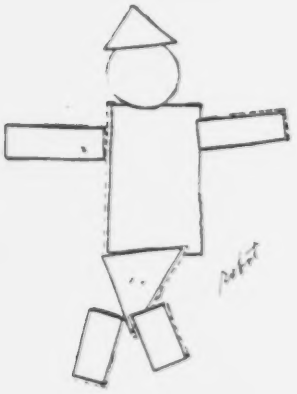
Jenny 3rd grade
three.
This is a house, the sun and arm
Picture #1

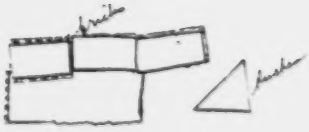


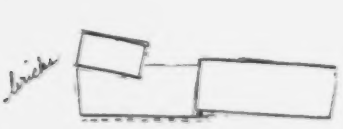
Jenny - 3rd grade
Picture #2

Picture #3

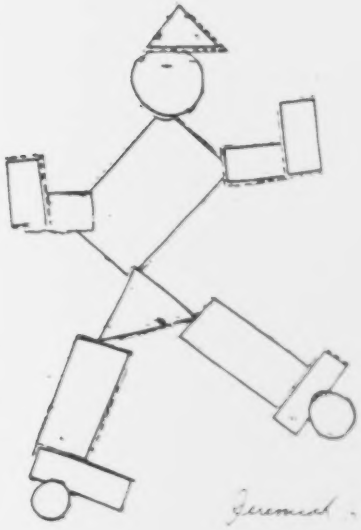




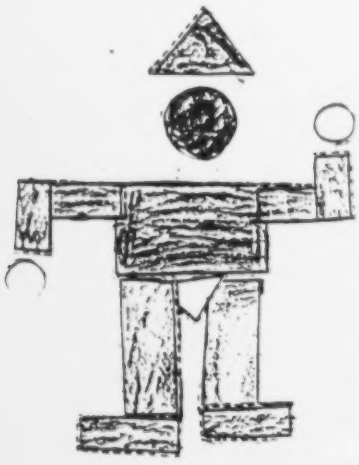




a Monster Robot
a scarecrow robot



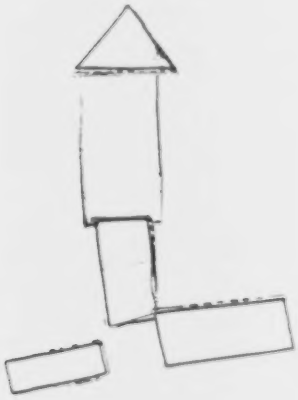
This is Thetaka the Clown and he lives in Illinois he goes to Shyanin school and he is a boy sometimes he likes to act in drama school



Jeremiah

This is by Jim Thompson Illinois
3rd grader

a stage

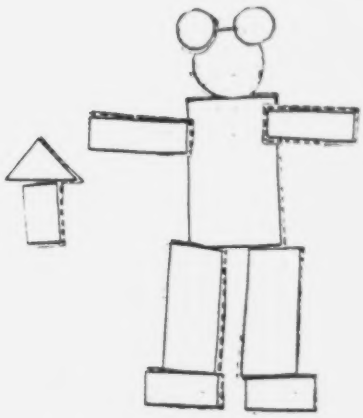


Sergeant - 3rd grader

Explained & was all the clowns
Gave to him - shown them
In the school and he
said I read them

Mickeymouse
in
the forest

Picture #1

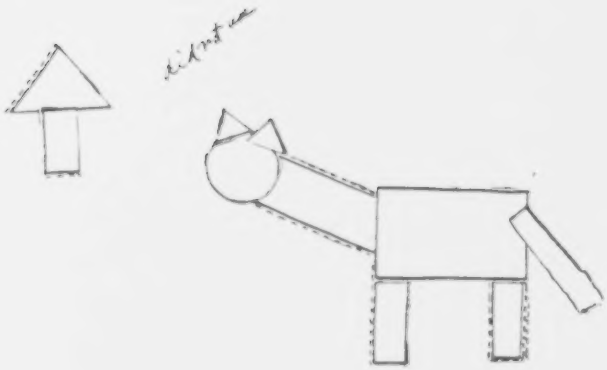


Sketches 5th grade

Giraffe

In the Zoo

Picture #2



Sketches 5th grade

WHAT, NO BOOKS? - HOW ABOUT DOMINOES?

Jean Anderson

After participating in the Summer Math Institute, I was eager to begin math with my 3rd Grade class. Knowing that "summer loss" usually necessitates review, I planned to begin with simple addition, subtraction, number comparison and place value. I decided that a couple of sets of dominoes would help with all these math needs.

"Don't get out your math books, today."

"But, aren't we doing math?"

Telling the students they didn't need their textbooks threw them at first, but they soon found out what was going on. Walking around the room, I gave each student a domino and asked what math they could do with their domino. Responses included counting the dots and adding the top and bottom numbers. We discussed the idea that counting the dots and adding the top and bottom was about the same. You put one number in your head and add-on or count-on the other number. While we were trading dominoes and writing down simple addition problems, one young man said, "You can turn it upside down and get another problem."

"Is it really another problem?" We discussed 3 + 5 = 8 & 5 + 3 = 8

Are they the same?

Another student said, "Mine equals seven." This caused others to say, "No, I have seven." So, we discussed the different combinations that would equal a certain sum. 3 + 4 = 7, 5 + 2 = 7, 6 + 1 + 7.

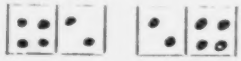
We moved from the simple addition to comparing numbers on either end of the domino, that is, finding the greater and subtracting the smaller. This led to a lesson on finding number families. Each child was asked to produce four problems, two addition and two subtraction, using the dots on their domino. (I excluded the dominoes with blanks)

Thus, a child with would generate:

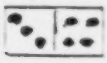
5	6	11	11
±6	±5	-6	-5
11	11	5	6

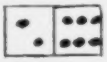
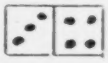
By passing dominoes around the room, the students were able to do many problems "without using a textbook."

After a brief review of place value for ones and tens, we turned our dominoes sideways and produced 4 2 or 2 4.



Again, we discussed greater and less than. (42 being greater than 24) A further discussion of hundreds and thousand places led to placing two dominoes side-by-side and calling out number names.

3 4  6 2 Are there any other combinations?

4 3 6 2 4 3 2 6  3 4 

Of course, someone placed several dominoes end-on-end, and we discussed other place values beyond the thousands.

The dominoes have momentarily disappeared to the closet shelf, but who knows when they may return? We have experimented and investigated with dice and a deck of playing cards. We have marked off rectangles on the floor with tape and have physically become the dots on the ends of dominoes.

No one bothers to ask, "How can we have math if we don't open our books." They just wait to see what happens!!

CUTTINGS

Teachers who come to the Teacher's Center turn in a weekly diary. In it they've written about what they've been reading, activities they've tried with kids and their observations of children's thinking along with general comments they wish to make about what they've been thinking, reading, doing and observing.

Many of these diaries contain material that is too good to remain buried in someone's file; it begs to be shared. So, in "Cuttings" we've simply lifted parts of individual diaries and reprinted them. Read them, think about them,. share them and help them grow.

CUTTINGS
Linda Kahley

Based upon **SEEDBED 24 : BRITISH EYES ON AMERICAN MATH** An interview with O'Brien, Lingard, and Johnson

The British assessment procedures are quite different from those we in the U.S. employ. They evaluate a student over a period of several years, in order to evaluate long-term growth and development. There are no multiple choice tests because they don't reflect the thought process of the student, and the thought process is more highly valued than is the "correct answer". The teachers share input into the evaluation of a student, as well as standardize the criteria used in the assignment of grades.

I like the idea of a teacher being directly involved with a child for a period of years. What better way to really assess a child's long-term progress? The children and the teachers both experience more continuity in the education process. Also, it could have the added benefit of removing some of the pressure on teachers to, "present all the concepts; to drill them in, so next year's teachers will think I did a good job with my students." We tend to overevaluate; to stress the product; the output more than the process. It is as if the teacher were the quality control inspector at a factory. One is constantly checking the products (the children) and therefore has little time to direct the process.

Teachers as professionals, capable of devising and distributing curriculum materials, capable of making valid assessment tools of their own!!? What a revolutionary idea. How could American teachers possibly have time (even if they had permission) to do those things and still keep the "paper mill" in motion?

The British emphasis on problem solving and actually devising and solving real, concrete problems seems to develop minds which are free and unfettered. It is important to be able to make guesses and to find workable alternative solutions. Lingard and Johnson's suggestion to use calculators for

arithmetic skills might not be widely accepted here. It seems, though, that their use of calculators is coupled with increasing ability and willingness on the part of their students to make predictions. One would make an estimate and then use the calculator or confirm or disprove that prediction.

I agree with the idea of "reducing the quantity and improving the quality" of education. Instead of teaching and reteaching the same things year after year, maybe we should slow down; teach fewer things well and let the students manipulate the concepts until they become internalized, and then build upon their knowledge base. Teaching "fractions" or "telling time" to a child means very little if the child doesn't understand the concepts. It can be likened to exposing a child to chicken pox in the hopes that he will catch it and get a full blown case of chicken pox (develop understanding); maybe he'll get a light case (be able to spout back the language of the concept without really understanding); or maybe he won't catch it at all (totally miss the concept). In any case the teacher will have fulfilled the requirement to introduce the material.

Additionally, I noticed Lingard's description of his duties as head master did not include the term "administrator". He described himself as a "facilitator"; for the adult staff members; for the children in the school; and of the curriculum. He seemed to be genuinely interested in the growth of all those around him.

CUTTINGS

Diane Wooters

I have been reading with great interest, **PIAGET'S THEORY OF INTELLECTUAL DEVELOPMENT**, by Herbert Ginsburg and Sylvia Opper. I am beginning to understand the significance of the growth process taking place in the young child, particularly during the transition from the preoperational stage of cognitive development to the concrete operations state.

In trying to relate this to the classroom, I can see that the traditional first and second grade classroom force feeds concepts that the child may not be ready to understand. Since the age of seven is such a pivotal point in development, I think it would be fairer to withhold any drastic measures of evaluation until this crucial point is worked through. Understanding the development of children can certainly make a difference in evaluating them. I would hate to think that I, or someone else, might retain a second grader when perhaps the following year, the child may have caught up and done just fine.

CUTTINGS
Lisa M. Laing

Combinations: The child-watching task for this week was to present the child with the question, "If you have three different shirts and two different pairs of pants, how many days can you wear a different combination of shirts and pants."

I asked Corey, age 8, to help me with this task. I first placed in front of him 3 pairs of construction paper pants and 2 shirts. His first response was that there were 2 different outfits, but he quickly changed his answer to "3" after I asked him some questions to get him thinking about his answer. Next, I gave him the combinations of four pairs of pants and three shirts. He began to move around the shirt and pants, counting combinations as he went, but, after masking 10 combinations, he said, "There's a bunch!" He was satisfied with his answer and wasn't bothered by the fact that he didn't come up with a concrete answer. On this last manipulation, Corey was unable to change the combinations systematically. The number of shirts and pants was too great for him to gain control. I wanted to see if he understood some sort of system for discovering his answers, so I next presented him with two pairs of pants and one shirt. He quickly, and with confidence, told me there were 2 combinations and he showed them to me. Next, I gave him 2 pairs of pants and 2 shirts, and he quickly answered that there were 4 outfits. (He used the words "outfits" and "combinations" interchangeably.) Finally, I presented him with 2 pairs of pants and 3 shirts. He began to move them around (again with no apparent system) and answered that there were only four different outfits. He was quite sure of himself even after I pointed out that the last combinations (two pairs of pants and two shirts) had produced 4 outfits. He said, "Oh yeah. There's 4 here too."

CUTTINGS
Linda Kahley

Based Upon: "**The Child as a Scientist**", An interview with Mimi Sinclair

Professor Sinclair, a researcher in the development of acquisition of language, confirms most of the research findings I have read recently about children making order out of their world. She says that children are scientists from early infancy. My understanding of her term "scientist" is that the child is striving to be a "meaning maker" from his earliest days.

She comments on young children's attempts to fit a small cube into a larger one unsuccessfully, then putting the smaller one into their mouth and then their fingers into the larger one to confirm their perception that these are things which will go into and hold other objects. This process of verification goes largely unnoticed by most of the adult world and is often discouraged by remarks warning children to keep things out of their mouths.

The remarks about certain types of education harming the theory building tendency and even the confidence of children are both depressing and frightening. I have often observed the tendency of children as they get older to wait for adults (teachers and others) to prepackage things for them to be put together as directed. I think this tendency is also frightening as it's an attempt, albeit unintentional, to robotize our children, to make them all think the same way, and to stifle creativity. Even in recreational activities children seem incapable of planning their own activities without adult direction or intervention.

I agree with Prof. Sinclair that it is the teacher's job to arrange things in such a way that the child's verification of his theories will neither be thwarted or frustrated. That's a fine line to walk, but it is crucial for the child's total development. The crux of the problem seems to be in the attitude toward the mistakes of the children. As I have read in other research, the mistakes must be viewed as a "natural part of growing and learning; not as failed teaching or thickheaded children." I also agree that the child's performance should be evaluated holistically; not upon isolated events.

CUTTINGS

Beth Jennings

This week I tried a new way of subtracting with my sons as my child-watching task.

My 10 year old son really thought it was "neat." I found his comments interesting. (He felt it was easier to think in groups of 10) He was real curious and asked if the teachers at school knew about this. Then he wanted to know if he would get in trouble if he tried it at school. This is TERRIBLE that a child would feel he would get in trouble for using a method to get the correct answer because it is not the way the teacher solves the problem. I was surprised at how quickly he picked up this method when I think back to how long it took him to learn to borrow.

When I showed the method to my 13 year old son he wanted to try some more problems. He made up his own problems and worked them to come up with the correct answer using the new procedure. He just couldn't believe that it always worked. Then he asked me if this new way was an easier way to subtract. I told him that he would have to decide that for himself. His comment then was "this is just adding and that is how I do it the other way." His next comment was "would I get in trouble if I did this at school?"

Both boys worried about getting in trouble at school even though they were solving the problems and arriving at the correct answers. We program kids to do things in a certain way that even if they correctly solve a problem they are taught it is wrong if it is not computed in the teacher's method.

The more I think about how the American Education System is set up, the more I realize changes need to be made. I hope that after seeing this I can be a teacher that makes a difference. Child-watching is a very interesting and valuable activity for all teachers.

(The "new way" referred to above is subtraction without borrowing.
For example, I have

$$\begin{array}{r} 24 \\ \underline{-17} \end{array}$$

I can't take 7 from 4 and get a whole number so I'll take it from one of the two tens.

$$\begin{array}{r} 1 \\ 24 \\ \underline{-17} \end{array}$$

7 from 10 leaves 3 and three plus 4 is 7 so 7 is my answer. B.G.- editor)

CUTTINGS

Steven Suess

I found several chapters in **NOTES ON MATHEMATICS IN PRIMARY SCHOOLS** (Association of the Teachers of Math) and **YOUNG CHILDREN REINVENT ARITHMETIC** (Kamii and DeClark) that were both interesting and beneficial to me. I preferred the Kamii book and especially found reassurance in chapters 9 and 10 by DeClark, a bona fide classroom teacher. I have already shared several of her games with math teachers in my building.

Much of the "nuts and bolts" of stimulating numerical thinking is evidenced in chapter 7 which deals with activities and "do's" and "don'ts" for the instructor.

Perhaps this book more than any other helped me to realize that youngsters do not learn arithmetic (Piaget); rather, they construct it from within. For themselves as individuals, they reinvent it in a way that has meaning to/for them.

CUTTINGS
Linda Kahley

Based Upon: **MATHEMATICS IN PRIMARY SCHOOLS** Council Curriculum Bulletin Number 1972: Chapter 2

The teacher's role seems to be one of an active observer. She seems to have time to watch each child and to observe the discoveries he is nearing. This is crucial in order to be able to ask the students the proper question at the right moments to help him cross the line into discoveries.

The use of object boxes was an intriguing idea. The children have a responsibility in their own learning and so many concepts are laid before the children in this manner: the concept of quantity (though not tied directly to a particular object); place value; the ordinal numbers and counting. All the experiences described are just that, "experiences" done within the context of real life, therefore with obvious practical application. It is use of "teachable moments" rather than teaching concepts. The idea of numbering chairs to be arranged for a school program and then using those chairs to let the children "see" number relationships is fabulous. How many of these golden opportunities would be available in my "someday" classroom if I were alert to them. I'm sure the teacher planned the activity because there were number cards prepared for the chairs, but it was a completely usable lesson derived from a necessary task.

This school environment seems so activity oriented rather than just "busywork" oriented. The children's minds seem to be open and searching and capable of problem solving. The children's solutions for taking attendance with blue and white beads in groups of ten was astounding. I think this freedom to think and develop solutions is an invaluable part of education.

The children seem to have such a capacity for the discoveries, which, I suppose, is in direct correlation to their natural curiosity. However, it is obvious that the teacher must be there to provide the language of the concepts and to guide the students deeper into their discoveries.

This reading makes me wonder if our system of education has not squelched the natural inquisitiveness of children by trying to fit them into the school mold rather than having the school molded to fit the children as this English report suggests.

CUTTINGS
Barb Madden

EDUCATION AND THE CULT OF EFFICIENCY Raymond E. Callahan, University of Chicago Press, 1962.

I think Callahan said it all in the preface: *"...what has happened in American education since 1900 can be explained on the basis of the extreme vulnerability of our schoolmen to public criticism and pressure and that this vulnerability is built into our pattern of local support and control." (p. VIII)*

The establishment and control of public schools is primarily the responsibility of the state. To respond to the desire for local control, local school boards, representative of the community, establish policy for the school. Courts and their decisions, and the federal government also influence school decisions. One must ask, "Where is the influence of teachers?"

Callahan makes clear in the first three chapters the god that business had become in this country by the early 1900's. Men like Frederick Taylor studied business practices to determine ways to increase efficiency. If these means would increase business efficiency, it was only reasonable that they would also increase school efficiency. After all, in many communities then and now, in dollars, school really is the biggest business. There were further demands on the schools to teach the value of business, and to offer courses which would prepare students for a life in the business world.

As I read this very detailed book, I couldn't help but contrast the attitude toward teachers and students in **SEEDBED #24** on the British school system, and in Frederick Taylor's business society. To Taylor, man was a machine to be exploited. In the British schools teachers and students are unique individuals to be respected.

Finally, the complaints against the schools in the early 1900's could have been lifted from the 1983 report "**A Nation at Risk**" -- decline in standardized test scores; functionally illiterate high school graduates; diluted curriculum; inadequate teachers...

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